Media Provision With Outsourced Content Production*

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Abstract

I use a model with a platform facilitating interaction between consumers, advertisers, and content creators to explore the effects of introducing a subscription which allows consumers to avoid ads. The subscription increases provision of niche content, but increased advertising and a high subscription price may reduce the welfare of consumers who enjoy mass market content. The effect on total welfare depends on how much the platform increases payments to content creators as a result of the subscription.

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1 Introduction

The ability to stream media via the internet has facilitated the creation and dissemination of many new types of content. The new forms of content range from the extremely popular, such as video makeup tutorials, to extremely niche concepts like speedruns[1]. Part of the

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1. Speedruns are videos, usually live-streamed, of a player attempting to complete a computer game as quickly as possible.

reason for this rise in variety is that with streaming, each consumer on a platform can watch content which differs from that viewed by every other consumer. A platform does not have to sacrifice mass market consumers when adding niche content. Another reason for this rise is a democratization of content production. Platforms such as YouTube or Twitch host content created by third parties in exchange for a share of the revenue generated by that content. This business model lowers the barrier to entry for creators with innovative ideas.

If content ideas arrive randomly within a large population, then it makes little sense for a platform like YouTube to hire creators within the firm[2]. Creating a medium with low barriers for independent creators to post content allows the platform to avoid the costs of producing content itself. The downside is that in order to add this content, the platform must attract the content creator who makes it. This creates an effect that has heretofore been largely underexplored in the theoretical literature: attracting new content creators allows the platform to provide new varieties of content, and the new content expands the set of consumers interested in the platform which, in turn, will attract more revenue.

I present a model where a monopoly platform brings together content creators, advertisers, and consumers. The influence of each group on the others is shown in Figure 1. Consumers can view content in exchange for either a subscription fee or watching advertising. The creators provide the content on which advertisers advertise in exchange for a share of the subscription or ad revenue from consumers attracted by their content. Advertisers purchase ad views in order to generate trade with consumers, but the presence of the ads imposes a nuisance cost on the consumers. The platform acts as a clearinghouse which matches consumers to content creators and advertisers to content. It decides what advertising level to set, how much to charge for ads and the subscription, and how much revenue to share with the content creators. Because the motivating example for this project is online video, I refer to the general action of consuming content as "watching".

[Figure 1 goes here]

2. I thank Jim Peck for identifying this factor.

The objective of this article is to examine the welfare effects of a subscription which allows consumers to avoid advertising in the context of a three-sided market[3]. I begin with a highly simplified baseline model where consumers' gross utility of watching is homogeneous and advertisers' value per ad is homogeneous in order to focus on the implications of adding content creators to a multi-sided media market. I compare an equilibrium where the platform can offer an ad-avoidance subscription to one where it cannot.

When the platform increases the advertising level in the model without the subscription, the consumers who watch see more ads, but some consumers choose to leave the platform due to the increase in nuisance. The platform chooses the advertising level which maximizes the total number of ad views. If the platform can offer the subscription, then the consumers who are most sensitive to advertising choose to subscribe, and the platform's tradeoff when increasing the advertising level is reduced or eliminated. The subscription gives consumers the ability to avoid advertising so more consumers will watch in the equilibrium when it is available. The content creators receive a payment from the platform based on the number of consumers who watch their content. In the advertising-only equilibrium only creators with a relatively large audience size receive enough revenue to cover the costs of producing content. Because the subscription increases both viewership and revenue per consumer, the platform increases payments to creators in order to attract relatively niche content creators who in turn attract consumers with niche tastes.

The subscription transforms consumers' nuisance cost from advertising (which is purely wasteful) into a transfer to the platform. Additionally, the new creators are at least weakly better off with the subscription as a result of producing content, so the subscription will increase total welfare in most circumstances. The profit of the platform and creators always increases, but the effect on consumer welfare is more nuanced. Consumers attracted to the platform by new content creators who produce only when the subscription is present are have greater welfare with the subscription. Low nuisance cost consumers who enjoy content

3. While the "three-sidedness" is admittedly limited in the baseline model, the basic intuition behind my results survives extension to more complex specifications.

which would be created regardless of the presence of the subscription are either exposed to more advertising than in the advertising-only equilibrium or the cost of the subscription fee is higher than the nuisance cost they would receive in the advertising-only equilibrium. On the other hand, the consumers with a high nuisance cost who enjoy content which would be produced in either equilibrium are better off with the subscription because the price of the subscription is below the nuisance they would have borne in the advertising-only equilibrium.

The potential for market expansion after the addition of new content creates a tradeoff, albeit an imperfect one, between the welfare of consumers who enjoy mass market media and those who prefer more niche media. If the platform's ability to capture consumers' value increases whether through advertising or the subscription, then consumers who watch content on the platform are by definition worse off, but the value of additional content to the platform increases, meaning that it increases creator payments. Higher payments attract creators with smaller audiences and, as discussed with the subscription above, the new consumers attracted by this content are better off for having new media to watch. This tradeoff is imperfect because the platform is capturing additional value from the niche consumers as well as the mass market consumers.

Finally, market expansion from new content creators is also significant when considering the effect of the subscription on advertisers' profits[4]. Advertising in this model is informative, with each ad view informing the consumer about the existence of an advertiser's product and generating the possibility of trade. Intuitively, one would expect that the subscription would make advertisers worse off because they are able to reach fewer consumers and thus generate fewer trades. But the platform wants to increase the advertising level to drive more subscriptions, so it reduces the price of advertising. Advertiser profits may increase even though they are trading with fewer consumers. Additionally, the ad views lost to the subscription are compensated for by the presence of ad-viewing consumers on the new content markets, so the number of ad views and trades with consumers may *increase* as a result of

4. This point primarily applies to Section 4 because the platform is able to extract all advertiser value in the baseline model, but the important elements are present in the baseline comparison as well.

the subscription.

Previous Literature

A number of models have considered the incorporation of multiple revenue sources (e.g. Anderson and Coate (2005), Choi (2006), Peitz and Valletti (2008), and Crampes, Haritchabalet, and Jullien (2009)) but the majority of them have considered subscription revenue as a fee to be charged in addition to or instead of receiving advertising revenue. Previous models featuring ad-avoidance subscriptions (e.g. Tåg (2009) or Prasad, Mahajan, and Bronnenberg (2003)) have highlighted the fact that the subscription gives the platform the ability to extract value directly from consumers rather than relying on the value the advertisers place on advertising, reducing net consumer surplus (Tåg in particular discusses this effect in detail). Although this factor is present in my model, it is counterbalanced by increased welfare of consumers attracted by new content creators who come to the platform as a result of the subscription. Therefore the net effect of these two factors on consumer welfare is ambiguous because the addition of the content creators makes the subscription much more favorable to consumers in my model than similar subscriptions described in these earlier articles.

As Crampes, Haritchabalet, and Jullien (2009) discuss, platforms in most two sided models have a limited capacity to relay content which stems from the assumption that the platform has only a single distribution channel. If it wishes to add more of one type of content it must reduce provision of a different type. The key difference in my model is that I emulate platforms where this restriction does not apply[5]. Although a few articles have considered single firms with multiple channels for content provision (most notably Anderson and Coate (2005)) while others have considered horizontal differentiation in content provision(e.g. Anderson and Gans (2011)), I am unaware of any that have allowed the number of channels

5. Although this framework is most applicable to three sided platforms like YouTube and Twitch because of the niche nature of independently created content, many of the conclusions in this article extend to a cable provider adding channels to its television package or a streaming firm such as Netflix purchasing additional content at a cost. See Hagiu and Wright (2015) for a discussion of the tradeoffs involved between providing a service as a platform and direct provision.

to be an endogenous decision to the extent this article does. There have been other papers which acknowledge the importance of the supply side for content provision in other contexts. Maldonado (2017) conducts an empirical investigation into the effects of Twitch.tv's favorable treatment of large creators, but he is mainly concerned with using that platform's unequal treatment to explore the implications of content neutrality on networks, and does not explore the welfare implications of this market size effect. Crampes, Haritchabalet, and Jullien (2009) and Choi (2006) both consider media markets with free entry, but in both cases the markets consist of a number of single channel two-sided platforms competing with each other rather than a monopoly platform adding channels. If entry consists of multiple platforms competing then it will be in each platform's interest to discourage other platforms from entering. If instead a monopoly platform is hosting all of the new content, then attracting additional content creators is beneficial to the platform.

I am not aware of any other theoretical work which explicitly models a third side in a platform market, attracting content creators is mechanically similar to other models where a platform invests in a form of "first party content" (to use Hagiu and Spulber's (2013) terminology) to attract consumers. The platform uses content to compensate viewers for watching ads, and the more content it hosts the more viewers it can attract. A common theme in this variety of model is that the investment compensates consumers for a monetization method which damages the product in some way. Google's revenue for example, comes from diverting search toward sponsored results which may be less relevant than the nearby "organic" search results, so in order to attract the consumer the search engine must provide high quality organic results (Hagiu and Jullien 2011; White 2013). My model is similar in the advertising only equilibrium but (as Tåg (2009) emphasizes) the addition of the subscription also adds an element of price discrimination because consumers are able to choose between the "damaged" (ad supported) version of the service and using the undamaged version for a fee. This discrimination increases both investment and the extent to which the service is damaged.

The combination of investment and price discrimination makes this article similar to Alexandrov and Deb (2012), but with a significant difference in that increased ”investment” bears more resemblance to capacity investment than quality investment. The subscription in my model does not increase the quality of the service received by consumers in the intensive markets, so while price discrimination can create a Pareto improvement in Alexandrov and Deb's model, there will always be consumers who are worse off as a result of the subscription here.

The use of an ad-avoidance subscription invites comparison to the ad-avoidance literature, but most of the previous literature on ad-avoidance has assumed that the avoidance technology is external to the platform. Anderson and Gans (2011) find that the introduction of ad-avoidance increases the advertising level, consistent with the findings of this article. However they also find that ad-avoidance reduces the platform's ability to extract value from viewers of niche content, causing the platform to shift to greater provision of mass market media. In contrast, in my model the subscription fee allows the platform to capture viewer surplus *more* effectively and makes production of niche content *more* worthwhile. The major reason for this difference is that when the ad-avoidance technology is separate from the platform, the costs consumers pay to avoid advertising are purely deadweight loss. In my model, the costs become revenue for the platform/content creators and the ad-avoidance allows consumers to avoid the deadweight loss of nuisance costs which are in excess of the advertiser's value of an ad view. Therefore, increased avoidance is generally beneficial for total surplus.

2 The Baseline Model

The set of network externalities is complex with a three sided market. Therefore, to focus on the effects of adding the content creators, I begin with a baseline model using strong simplifying assumptions. Namely, homogeneous gross utility of watching for consumers and homogeneous advertisers. The main results from the baseline model are robust to relaxing these assumptions as explored in Section 4.

Baseline with no subscription

The structure of the content space is shown in Figure 2. There is a continuum of content types (I use the terms ”content type” and ”content market” interchangably) indexed by $t \in [0, \bar{t}]$ where $\bar{t}$ can be $+\infty$. Each consumer is exogenously assigned precisely one type of content and they only enjoy that type (similar to Bergemann and Bonatti (2011) or more broadly Yang (2013)). Each content type holds only one content creator and has a market size $g(t)$ which denotes the mass of consumers in that market. Because each type has only one creator, t is used to index creators as well as types. Without loss of generality, I assume the content types are ordered so that the market size is monotonically decreasing, and I make the technical assumption that $g(\cdot)$ is differentiable and invertable. Advertisers purchase ads which are disseminated among views on all markets according to the advertising level chosen by the platform.

[Figure 2 goes here]

Consumers

Without the subscription, consumers' only choice is whether to watch or not. Consumer utility is 0 if they choose not to watch and utility if they do watch is given by

$$u - a\eta \tag{1}$$

Gross utility of watching is $u > 0$ if the content creator for that type decides to produce and 0 otherwise. Consumers within a given content type are differentiated by their nuisance cost

$\eta > 0$ from advertising[6]. $\eta \in [\underline{\eta}, \bar{\eta}]$ is exogenously assigned and distributed across consumers independently from content type according to the log concave distribution function[7] $f(\eta)$. If $\bar{\eta} < u$ then all consumers watch at every advertising level, meaning that the platform would not face a meaningful decision when changing the advertising level. Hence for the rest of the article I assume $\bar{\eta} > u$. The nuisance cost faced by consumers is scaled by the advertising level $a \in [0, 1]$ set by the platform. For the purposes of this article, I view a as the mass of firms allowed by the platform to purchase advertising, each of whom reaches every watching consumer with a single ad with probability 1[8].

Consumers choose to watch if they receive weakly positive utility from doing so. I assume that indifferent consumers watch. Because the utility of the outside option is equal to 0 it is easy to see that consumers will watch only if the creator of their type is producing content (so the gross utility of watching is positive) and

$$\begin{aligned} & u - a\eta \geq 0 \\ \implies & \frac{u}{a} \geq \eta \end{aligned} \tag{2}$$

so the critical η below which consumers will choose to watch is $\frac{u}{a}$ and the proportion of consumers who watch in equilibrium in the advertising-only model is $F(\frac{u}{a})$, where $F(\eta) = \int_{\underline{\eta}}^{\eta} f(n)dn$. This proportion is constant across markets due to the independence of $g(\cdot)$ and

6. The assumption that all consumers find advertising to be strictly harmful net of trade is potentially contentious (See Crampes, Haritchabalet, and Jullien (2009) for a discussion of the implications of positive advertising externalities) but one that is supported empirically by the findings of Wilbur (2008) among many others. Even so, adding a set of consumers with negative nuisance costs would not substantially change my results because these consumers would be unresponsive to both the presence of the subscription and changes in the advertising level.

7. The total measure of consumers can be greater than 1 when summed across content markets. $f(\cdot)$ should be thought of as a population distribution.

8. Most online videos feature only a single advertisement before viewing. However, many longer videos also feature mid-roll ads, and shorter videos will often have display advertising. There are often ads next to the video or inserted between the video and its description, and the intrusiveness of the ads themselves can vary. Some ads allow viewers to opt out after a few seconds, whereas others run for as long as several minutes without the option to skip them. Therefore this continuous representation of the amount of advertising viewed by a consumer is more appropriate than it might initially appear.

$f(\cdot)$.

Content Creators

Content creators receive a payment from the platform per ad view. Creator profit is based on the total number of ad views their content receives and the fee set by the platform. The creators face only one decision: produce or not. If they choose not to produce they receive a payoff of 0. If they do produce they face a cost c and receive a payment w_a from the platform per ad view on their content. Given a market size $g(t)$, viewership proportion $F(\frac{u}{a})$, and advertising level a, total ad views will be $aF(\frac{u}{a})g(t)$, so the profit on production with advertising only is

$$w_a aF(\frac{u}{a})g(t) - c \tag{3}$$

Content creators produce if they earn weakly positive profit from doing so. This means that there is a critical $g(t)$ above which the content creator will produce. Setting the above expression equal to 0, it is then simple to solve for the creator who is indifferent between producing and not in equilibrium:

$$t^* = g^{-1}\left(\frac{c}{w_a aF(\frac{u}{a})}\right) \tag{4}$$

t^* denotes the critical content type. This is the content market where $g(t^*)$ times the average creator revenue per consumer is exactly equal to the cost of production. All creators with larger audiences will produce and no creator with a smaller audience will make content.

Advertisers

In order to focus on the implications of adding the content creator side to the market, I assume a particularly simple advertising sector for the baseline model. Advertisers have value z per ad view, which reflects the expected surplus from trade. All consumers have the same value for trade, so the advertisers can extract full value from the consumers and consumers receive 0 surplus from trade[9][10]. The platform charges a price p_a per ad view. The advertiser value per ad is then

$$\pi_{ad} = z - p_a \tag{5}$$

Because advertisers are homogeneous, the platform sets $p_a = z$ and extract all of the surplus from the advertisers.

The Platform

The platform earns profit on the margin between the fee it receives per ad view and the creator payment per ad view. The platform's total profits will be the sum of these margins across all ad views. It will select $p_a, a,$ and w_a to maximize

$$\pi_{plat} = (p_a - w_a)aF(\frac{u}{a})G(t^*) \tag{6}$$

Where

$$G(t^*) = \int_0^{t^*} g(t)dt \tag{7}$$

p_a was determined in the advertiser's problem, so all that remains is to evaluate the platform's choice of a and w_a. I impose the following assumption:

Assumption 1. *$F(\eta) - f(\eta)\eta$ is strictly increasing and crosses from negative to positive exactly once as η increases.*

9. This is the setting of Anderson and Coate (2005). Alternatively one can think of the nuisance cost as being net of expected trade surplus. My results do not change so long as the total effect of advertising on utility is negative.

10. This linear returns advertising technology is similar to Choi (2006). Although Section 4 shows that the homegeneity assumption has a significant impact in this model, changing the returns to scale on advertising would not significantly change my main results.

Assumption 1 is not equivalent to log-concavity (for example, the uniform distribution does not satisfy it). For many log-concave distributions $F(\eta)-f(\eta)\eta$ is increasing in η, so this assumption is equivalent to assuming that $F(\underline{\eta}) < f(\underline{\eta})\underline{\eta}$ in those cases, but monotonicity is not necessary so long as this single crossing property is satisfied. This assumption is not strictly necessary for equilibrium; I give numerical examples of equilibria using distributions which do and do not satisfy this assumption later. It does ensure that the platform's advertising level problem has at most one solution for which the first order condition is sufficient. I use the accent "$\wedge$" to refer to the equilibrium values for endogenous variables when the subscription is unavailable. The equilibrium is given in the following lemma (all proofs are relegated to the appendix):

Lemma 1. *If $f(\cdot)$ satisfies Assumption 1 and $g(\cdot)$ is concave, then there is a unique equilibrium in the baseline model with no subscription. The price of advertising is $\hat{p}_a = z$. The advertising level solves Equation (8) if the solution can be reached with $\hat{a} \in (0, 1]$*

$$F(\frac{u}{\hat{a}}) = f(\frac{u}{\hat{a}})\frac{u}{\hat{a}} \tag{8}$$

Otherwise the equilibrium $\hat{a}$ will be 1. $\hat{w}_a$ solves Equation (9):

$$\frac{1}{g'(\hat{t}^*)}\frac{-c^2}{\hat{w}_a(\hat{w}_a\hat{a}F(\frac{u}{\hat{a}}))^2}(z-\hat{w}_a) = G(\hat{t}^*) \tag{9}$$

Note that $\hat{t}^*$ is fully determined by Lemma 1 and Equation (4). Equation (9) represents the platform's tradeoff between attracting additional creators and increasing the payment to existing creators. A solution to this first order condition always exists as no creators will produce when the payment is 0 and marginal benefit of additional markets is decreasing and reaches 0 as $\hat{w}_a$ approaches $\hat{p}_a$. Concavity of $g(x)$ ensures that platform profit is concave in the creator payment and gives sufficiency of the first order condition for profit maximization.

When the platform increases the advertising level, it increases the number of advertisements but drives away some consumers due to the increased nuisance cost. Equation (8) captures this tradeoff, and Assumption 1 ensures a unique solution which will maximize total ad views.

The platform and content creators' profits are both maximized when the number of ad views is maximized, so their interests are aligned when it comes to advertising levels, but the inability of the platform to set different payments for different creators means that it is paying the creators with the largest audiences too much from both a social welfare and profit maximizing perspective. If the platform were able to differentiate among creators when deciding revenue shares, then it could reduce per-view payments to creators with a large audience and attract more niche creators by increasing the payments on small markets until niche creators are just willing to produce content. Total welfare would increase because the audience of these niche creators must collectively value this additional content more than the cost of creating it. Otherwise the platform would not be able to extract enough revenue to be willing to pay the creator. Consumers of the mass market content would be no worse off.

Baseline with subscription

I now evaluate the baseline model when the platform can offer the subscription. Let p_s be the subscription price, utility from watching with the subscription is

$$u - p_s \tag{10}$$

u does not vary across consumers, so if $p_s > u$, no consumer purchases the subscription and the remaining decisions are equivalent to the baseline with no subscription. If $p_s \leq u$, then the minimum possible utility from watching when the relevant content creator produces is $u - p_s \geq 0$. This means that for $p_s \leq u$, all consumers watch if their associated creator produces content and their only relevant decision is whether to purchase the subscription.

Lemma 3 below shows that the equilibrium with the subscription is always more profitable than the one without, so the platform sets $p_s \leq u$ and the proportion of consumers who view content in the baseline model with the subscription is 1. Consumers purchase the subscription if

$$u - p_s \geq u - \eta a \implies \eta \geq \frac{p_s}{a} \tag{11}$$

This condition states that the subscription is more appealing than watching with ads if the price is less than or equal to the nuisance cost from advertising (assuming consumers subscribe when indifferent). The proportion of consumers who watch ads is $F(\frac{p_s}{a})$

Let w_s be the creator payment received per subscription view. Then creator revenue is now the revenue from advertising per viewer, plus the revenue from the subscription per viewer, multiplied by the number of viewers of each type. Therefore, creator profit when the subscription is available is given by

$$\left(w_a a F(\frac{p_s}{a}) + w_s(1 - F(\frac{p_s}{a}))\right) g(t) - c \tag{12}$$

and so the indifferent content creator in the model with the subscription is

$$t^* = g^{-1}\left(\frac{c}{w_a a F(\frac{p_s}{a}) + w_s(1 - F(\frac{p_s}{a}))}\right) \tag{13}$$

The platform's revenue with the new income stream is the weighted average of the margin on advertising views and the margin on subscription views, multiplied by the total number of views.

$$\pi_{plat} = \left[(p_a - w_a) a F(\frac{p_s}{a}) + (p_s - w_s)(1 - F(\frac{p_s}{a}))\right] G(t^*) \tag{14}$$

The advertiser's problem is unaffected by the presence of the subscription, so the price of advertising is still $p_a = z$. The platform has 4 undetermined decision variables: w_s, w_a, p_s, a.

Similar to the "$\wedge$" accent in the previous section, I use "$\sim$" to distinguish equilibrium values of endogenous variables when the subscription is available. To understand the platform's decision making, it is helpful to define $\tilde{p} = \tilde{p}_a \tilde{a} F(\frac{\tilde{p}_s}{\tilde{a}}) + \tilde{p}_s(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))$ and $\tilde{w} = \tilde{w}_a \tilde{a} F(\frac{\tilde{p}_s}{\tilde{a}}) + \tilde{w}_s(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))$ Then in the equilibrium with the subscription

$$\tilde{\pi}_{plat} = (\tilde{p} - \tilde{w})\, G(\tilde{t}^*) \tag{15}$$

and

$$\tilde{t}^* = g^{-1}\left(\frac{c}{\tilde{w}}\right) \tag{16}$$

$\tilde{p}$ is the platform's revenue per watching consumer and $\tilde{w}$ is the creators' revenue per consumer who watches their content. A lemma follows:

Lemma 2. *There exists a continuum of platform profit maximizing pairs $(\tilde{w}_s, \tilde{w}_a)$, but they are all equivalent to a single optimal value of $\tilde{w}$. This $\tilde{w}$ solves*

$$\frac{-c^2}{g'(\tilde{t}^*)} \frac{\tilde{p} - \tilde{w}}{\tilde{w}^3} = G(\tilde{t}^*) \tag{17}$$

If $g(\cdot)$ is concave, this solution exists and is unique for a given value of $\tilde{p}_a, \tilde{p}_s,$ and $\tilde{a}$.

The intuition behind this lemma is straightforward, if the platform increases $\tilde{w}_a$ by ϵ and decreases $\tilde{w}_s$ by $\frac{aF(\frac{\tilde{p}_s}{\tilde{a}})}{(1-F(\frac{\tilde{p}_s}{\tilde{a}}))}\epsilon$, then both producer and platform profits are unchanged. There is no point prediction for the creator payments, but rather a prediction that equates the marginal benefit of adding an additional market to the cost of giving existing producers a greater proportion of total revenue. The equilibrium in this market will then be given by the following lemma:

Lemma 3. *In the baseline model with the subscription available, the price of advertising is $\tilde{p}_a = z$. If Assumption 1 is satisfied and $g(\cdot)$ is concave then:*

1. *If $z > u$:*
   - $\tilde{p}_s = u$
   - $\hat{a} < \tilde{a}$ *and* $\tilde{a}$ *solves Equation* (18) *unless this solution would put it above the corner at 1, in which case* $\tilde{a} = 1$.

$$p_a \left(\frac{u}{a} f(\frac{u}{\tilde{a}}) - F(\frac{u}{\tilde{a}}) \right) = \left(\frac{u}{\tilde{a}} \right)^2 f(\frac{u}{\tilde{a}}) \tag{18}$$

2. *If $z < u$:*
   - $\tilde{a} = 1$
   - $\tilde{p}_s$ *solves*

$$\frac{1 - F(\frac{\tilde{p}_s}{\tilde{a}})}{f(\frac{\tilde{p}_s}{\tilde{a}})} = \tilde{p}_s - z \tag{19}$$

   if the solution is less than u, or $\tilde{p}_s = u$ if it is not.

3. $\tilde{w}$ *solves Equation* (17)

If $z > u$, then an advertising view is more valuable than a subscriber view, but a subscriber is more valuable than a consumer leaving the platform. The platform sets the highest subscription price where consumers will still purchase it to encourage only the consumers who would otherwise not watch to subscribe. The platform could set $\tilde{a} = \hat{a}$ and $\tilde{p}_s = u$, have the exact same number of ad viewing consumers as it had previously, and still make more profit than if there was no subscription due to new consumers joining the platform. However the addition of the subscription means that the opportunity cost of raising the advertising level is lower as consumers are switching to the subscription rather than away from the platform, so the profit maximizing advertising level is higher than when advertising is the only revenue stream.

When a subscription view is more valuable than an advertising view (i.e. when $u > z$), the platform's optimal advertising level is the corner solution. The subscription fee is always weakly below u, so increasing the advertising level drives additional subscriptions rather than driving away viewership. In this case, the opportunity cost of increasing the advertising level has not only gone down but disappeared entirely. Because subscription views are now more valuable, increasing the subscription price means trading off additional revenue from subscribers with lost subscriptions as marginal consumers move from the subscription to watching with ads. Equation (19) balances these effects.

The case for which $z > u$ does not seem to apply in actual media markets. Estimates for CPM (revenue per thousand ads) for YouTube can be as low as USD$2-$10 (Green 2015; Gutelle 2014) whereas YouTube's subscription service is approximately $10 per month at the time of writing, meaning that a free viewer would need to watch 1,000-5,000 ads per month to give as much revenue as a single subscriber. Comparable numbers hold for similar platforms like Twitch and Hulu, so for the remainder of this article I will focus primarily on equilibria where consumer value is significantly greater than advertiser value. This is formalized in Assumption 2:

Assumption 2. $z < u$

3 Welfare Comparison

Platform and creator profits unambiguously increase with the introduction of the subscription fee. When evaluating the effect on consumer welfare, it is helpful to separate the content markets which are served in both equilibria from the markets which are only served when the subscription is available. I therefore define the former markets as *intensive* markets and the latter as *extensive* markets.

Proposition 1. *If Assumption 1, Assumption 2 and concavity of $g(\cdot)$ are satisfied, then*

1. $\tilde{t}^* \geq \hat{t}^*$ *and creators receive more revenue when the subscription is available than when the subscription is not available.*

2. *Intensive market consumers for whom $\eta < \tilde{p}_s$ will watch content with ads in both the advertising-only equilibrium and the equilibrium with the subscription. Their net utility will be lower in the equilibrium with the subscription than in the advertising-only equilibrium.*

3. *Intensive market consumers for whom $\tilde{p}_s \leq \eta \leq \frac{\tilde{p}_s}{\hat{a}}$ will purchase the subscription when it is available, but their net utility of watching will be lower in the equilibrium with the subscription than in the advertising-only equilibrium.*

4. *Intensive market consumers for whom $\frac{\tilde{p}_s}{\hat{a}} < \eta$ purchase the subscription when it is available. Their net utility is greater in the equilibrium with the subscription than in the advertising-only equilibrium.*

5. *The net utility of extensive market consumers is at least weakly greater in the equilibrium with the subscription than in the advertising-only equilibrium.*

Creator profits increase when the subscription is available for two reasons: i.) The option to subscribe increases viewership so creators would get additional revenue even if $\tilde{w}$ were to remain constant, and ii.) The marginal value of an additional content market is larger with the subscription, meaning that the the platform will be willing to set $\tilde{w} > \hat{w}$ in order to serve more markets even though this means increasing payments to creators who already produce. Figure 3 shows market coverage in the two versions of the baseline model and provides a more detailed breakdown of the consumer groups from Proposition 1.

[Figure 3 goes here]

Figure 3a shows market capture in the advertising-only model. The vertical axis is the size of a given content market, but it is helpful to think of the consumers as being ordered in ascending η, so that consumers closest the horizontal axis have the lowest nuisance cost. Consumers above the $F(\frac{u}{\hat{a}})g(t)$ line find the nuisance cost too high to consider watching, and those to the right of $\hat{t}^*$ prefer content where too few consumers watch for the creator to produce. The consumers in the lower left region are both willing to watch and have content provided for them.

Figure 3b shows market coverage in the model with the addition of the subscription. All consumers who were watching previously continue to watch, but whether they benefit from the presence of the subscription depends on their nuisance cost. The highest nuisance cost consumers who were watching in the advertising-only equilibrium (those in Region 3) are have increased welfare with the subscription because the price is lower than the nuisance cost they would suffer if the subscription were unavailable. Consumers for whom $\tilde{p}_s \leq \eta \leq \frac{\tilde{p}_s}{\hat{a}}$ purchase the subscription, but pay more in the subscription fee than they would in nuisance cost in the advertising-only equilibrium. They are therefore worse off than they were in that equilibrium. These consumers lie in Region 2. Finally, the consumers who continue to watch with ads (Region 1) are worse off with the subscription available because they are viewing the same content with more advertising.

Two sets of new consumers begin watching content as a result of the subscription. Consumers who previously suffered too much nuisance cost to find the platform worthwhile now purchase the subscription and watch, these lie in Region 4. Additionally, because creators are both getting more views and receiving more revenue, more choose to create, which means consumers on previously un-served markets are now watching as well. This is the extensive change in market coverage and is represented by Region 5. All consumers who begin to watch as a result of the subscription when they would not in the advertising-only model are at least weakly better off with the subscription.

If the subscription price is such that $\tilde{p}_s > \frac{u}{\hat{a}}$ (which can occur if Assumption 2 is violated or if the mass of $f(\cdot)$ is sufficiently concentrated near $\bar{\eta}$) then Region 2 and Region 3 will not exist because the only consumers who purchase the subscription are those who did not watch when it was not available. The changes in aggregate welfare and how much of the benefit from the subscription is captured by the platform depends on $f(\cdot)$ and $g(\cdot)$.

Total Welfare

The platform captures all of the advertiser surplus and content creator revenue is a fraction of platform revenue. The total non-consumer welfare in the advertising-only model is

$$(\hat{p}-\hat{w})F(\frac{u}{\hat{a}})G(\hat{t}^*)+\hat{w}F(\frac{u}{\hat{a}})G(\hat{t}^*)-\hat{t}^*c=\hat{p}F(\frac{u}{\hat{a}})G(\hat{t}^*)-\hat{t}^*c \tag{20}$$

The first term on the left hand side represents platform revenue, the second the creator revenue, and the third the cost of production borne by the creators. Adding them together gives the total revenue minus the cost of production as shown on the right hand side. Similarly, non-consumer welfare in the model with the subscription is $\tilde{p}G(\tilde{t}^*)-\tilde{t}^*c$. Total welfare in the advertising-only model is

$$\left[z\hat{a}F(\frac{u}{\hat{a}})+\int_{\underline{\eta}}^{\frac{u}{\hat{a}}}(u-\hat{a}\eta)dF(\eta)\right]G(\hat{t}^*)-\hat{t}^*c \tag{21}$$

The first term in the brackets is non-consumer revenue, and the second is the average net utility of consumers. Adding them together gives the total social benefit to content production, and $\hat{t}^*c$ is the cost. Total welfare in the model with the subscription is

$$\left[\tilde{p}_aF(\tilde{p}_s)+u-\int_{\underline{\eta}}^{\tilde{p}_s}\eta dF(\eta)\right]G(\tilde{t}^*)-\tilde{t}^*c \tag{22}$$

With the addition of the subscription, subscribing consumers no longer bear the nuisance cost of advertising. This nuisance cost is always greater than the social benefit because the benefit of advertising per consumer is equal to $\tilde{p}_a = z$[11], and the consumers who subscribe are willing to pay $\tilde{p}_s > z$ to avoid the ad. The price of the subscription cancels out of total welfare because it is a transfer from consumers to the platform. Taking the difference of the two welfare equations, the change in total welfare is

$$\Delta\Omega = \underbrace{\left[z\left(F(\tilde{p}_s) - \hat{a}F(\frac{u}{\hat{a}})\right) + (1 - F(\frac{u}{\hat{a}}))u + \hat{a}\int_{\underline{\eta}}^{\frac{u}{\hat{a}}} \eta dF(\eta) - \int_{\underline{\eta}}^{\tilde{p}_s} \eta dF(\eta)\right] G(\hat{t}^*)}_{\text{Welfare change for content covered in both models}}$$
$$+ \underbrace{\left[zF(\tilde{p}_s) + u - \int_{\underline{\eta}}^{\tilde{p}_s} \eta dF(\eta)\right] (G(\tilde{t}^*) - G(\hat{t}^*)) - (\tilde{t}^* - \hat{t}^*)c}_{\text{Content produced with addition of subscription}} \tag{23}$$

The first term in $\Delta\Omega$ is the change in welfare on the intensive markets, which can be either positive or negative depending on whether the increased profits and ability to avoid advertising outweigh the losses for low nuisance cost consumers. The second and third terms are the extensive welfare change, which is strictly positive because all creators with market sizes larger than $g(\tilde{t}^*)$ make positive profits, and the benefits to consumers and the platform are non-negative. Although both the consumers from Region 1 and Region 2 in Figure 3b are worse off with the subscription, the Region 2 consumers are worse off because they are giving a substantial transfer to the platform in the form of a subscription fee, so their contribution to the change in total welfare is positive. Therefore, total welfare can only decrease with the subscription if the increase in nuisance costs faced by Region 1 consumers is large relative to the welfare change of all other groups in the model. The circumstances where this occurs will be rare as it requires a high subscription price (suggesting that the platform is able to extract a substantial amount of consumer surplus) and a small extensive change in market coverage (which would require that the platform does not extract much consumer surplus relative to the cost of acquiring new markets).

11. From Assumption 2, $\tilde{a} = 1$.

Platform and Creator Profits

Separating the welfare change into the change in platform/producer welfare and the change in consumer welfare. The change in platform/producer welfare is

$$
\begin{aligned}
\Delta\pi = & \underbrace{\left[z(F(\tilde{p}_s) - a * F(\frac{u}{\hat{a}})) + \tilde{p}_s(1 - F(\tilde{p}_s))\right] G(\hat{t}^*)}_{\text{Content covered in both models}} \\
& + \underbrace{\left[zF(\tilde{p}_s) + \tilde{p}_s(1 - F(\tilde{p}_s))\right](G(\tilde{t}^*) - G(\hat{t}^*)) + (\hat{t}^* - \tilde{t}^*)c}_{\text{Content produced with addition of subscription}}
\end{aligned}
\tag{24}
$$

Both the intensive and extensive changes are positive, so profit is unambiguously higher in the subscription model. This is sensible because the advertising-only structure remains available with the addition of the subscription, so if the subscription were not more profitable the platform would not offer it. This increase in profit is also necessary to have the extensive welfare change for consumers. Otherwise the platform would not increase creator payments and no new creators would choose to produce content.

Consumer Welfare

$$
\begin{aligned}
\Delta U = & \underbrace{\left[(1 - F(\frac{u}{\hat{a}}))u - (1 - F(\tilde{p}_s))\tilde{p}_s - \left(\int_{\underline{\eta}}^{\tilde{p}_s} \eta dF(\eta) - \hat{a}\int_{\underline{\eta}}^{\frac{u}{\hat{a}}} \eta dF(\eta)\right)\right] G(\hat{t}^*)}_{\text{Content covered in both models}} \\
& + \underbrace{\left[u - (1 - F(\tilde{p}_s))\tilde{p}_s - \int_{\underline{\eta}}^{\tilde{p}_s} \eta dF(\eta)\right](G(\tilde{t}^*) - G(\hat{t}^*))}_{\text{Content produced with addition of subscription}}
\end{aligned}
\tag{25}
$$

The intensive market change in consumer welfare (the first term on the right hand side of Equation (25)) has ambiguous sign. As previously described, low nuisance cost consumers are worse off and high nuisance cost consumers better off with the subscription. The aggregate change in intensive welfare depends on the distribution of η. If there is a relatively high mass of consumers with high nuisance costs, then the benefits of the subscription to the high nuisance cost consumers outweigh the losses of those with low nuisance costs, but if the proportion of high nuisance cost consumers is too high, then this effect is counteracted by a high subscription price and all of the benefits of the subscription are captured by the platform and creators. Aggregate intensive market consumer welfare decreases with the subscription if either the proportion of consumers who are highly averse to advertising is too low, or too high. The extensive market welfare change (the second term on the right hand side of Equation (25)) is positive because those content types were previously not produced so consumers can be no worse off than if there was no content. If the extensive change is sufficiently large, it may be enough to counteract a decrease in welfare on the intensive markets, raising aggregate consumer welfare even if the intensive market consumers are worse off with the subscription.

This highlights a tradeoff between the welfare of consumers on the intensive markets and the extensive market consumers' welfare. The higher the platform's profit, the more it is willing to pay creators and the greater the increase in content (i.e. the greater $G(\tilde{t}^*) - G(\hat{t}^*)$). However, once the additional consumers who join the platform with the addition of the subscription are accounted for, an increase in the platform's profit must imply an increased ability to extract value from consumers, so the consumers who would watch anyway benefit less from the subscription. This tradeoff is imperfect in that the more surplus the platform can extract, the less the extensive market consumers benefit from being served as more of their value is being captured by the platform.

As the mass of consumers with high nuisance cost increases, total surplus in the equilibrium with the subscription increases relative to advertising-only on all markets (including the intensive ones). As the size of the content markets becomes more even, t^* becomes more responsive to changes in w, which would increase the (positive) extensive change, but then $\hat{t}^*$ will be higher in this case as well, so relative size of the extensive change might decrease even though total coverage would be higher compared to a more concentrated market. Although an increase in total welfare from the subscription is the most likely outcome, it is possible for the subscription to reduce welfare if the increased nuisance costs from consumers who continue to watch ads outweighs the benefits to other agents. This can happen if consumers are highly insensitive to ads and the distribution of consumers is such that the size of the extensive market change is small compared to the intensive market consumers' welfare decrease.

Numerical Example

To demonstrate how changing the distributions changes the results, I compute two numerical examples using different nuisance cost distribution functions: a right triangular distribution, and a half-normal distribution. The triangular distribution has domain $[\underline{\eta}, \bar{\eta}]$ with mode $\underline{\eta}$. The density function is then $\frac{2(\bar{\eta}-\eta)}{(\bar{\eta}-\underline{\eta})^2}$. Although not often used, the triangular distribution is a good approximation of many distributions, and has the benefit of being analytically tractable for this model[12]. The half normal distribution (density function $2\phi(x), x \geq 0$ where $\phi(\cdot)$ is the standard normal pdf) is an example of a distribution where Assumption 1 does not hold, but an equilibrium exists despite this fact. With the half-normal distribution $F(\eta) - f(\eta)\eta$ is always positive, meaning that the platform will always set the maximal advertising level. Most of the results from earlier in the article are robust to this effect. Now set

$$g(t) = (1 - t^{(\alpha-1)}) \tag{26}$$

where $\alpha \geq 1$ is a concavity parameter for the market size function. The higher α, the less concave the function, meaning that t^* will be more responsive to changes in w[13]. I use this function for the example because it is tractable, and concavity is a function of a single parameter which facilitates demonstrating the effects of variation in curvature of market

12. Analytical solutions for the advertising level and subscription price with the triangular distribution are available in an online appendix on the author's website. Numerical calculations for both distributions were performed using Matlab R2016b. Code available from the author upon request.

13. In a slight abuse of notation, I use w to refer to creator payments when statements can refer to either equilibrium. Unaccented p should be interpreted similarly.

size[14].

[Figure 4 goes here]

The curves in Figure 4a show the variation in ΔU and $\Delta\Omega$ as $\bar{\eta}$ changes in the triangular distribution. As the consumers become more sensitive to advertising, the platform is able to capture more of their surplus (and so more of the benefits of the subscription) by increasing the subscription price. As $\bar{\eta}$ becomes sufficiently high, $\tilde{p}_s$ approaches the corner solution at $\tilde{p}_s = u = 1$ and all of the benefits of the subscription are captured by the platform. The kink in the curves occurs when the price reaches a corner. Eventually, all intensive consumers are weakly worse off with the subscription due to either the increase in nuisance cost from consumers who do not purchase the subscription or the high subscription cost capturing all of their surplus. The same holds true for subscribers on the extensive markets, but the low nuisance cost extensive market consumers who watch the content with ads still are strictly better off in the subscription equilibrium.

The welfare changes depicted in Figure 4b show that the half normal distribution is qualitatively different from the triangular distribution. Because $a = 1$ in both models the consumers all see a weak improvement to welfare with the presence of the subscription and are initially strictly better off as the average nuisance cost increases. When σ increases more consumers purchase the subscription, and the consumers who use the free version are no worse off because the advertising level was already as high as it could get, but as consumers become more sensitive to advertising, the subscription price increases and the platform begins to capture more consumer surplus, which cuts into the increase in consumer welfare from the subscription.

[Figure 5 goes here]

14. This function is not normalized to keep total consumer mass constant, but doing so produces qualitatively identical results.

Figure 5a shows the welfare changes as the market size function becomes less curved. The change in utility and the increase in welfare both become more intense[15] when nuisance costs follow a triangular distribution. In other parameter settings where the utility change is positive, it becomes more positive (albeit at a similarly shallow rate) as α increases. This is because as the market size curve becomes flatter, the marginal benefit of additional content markets becomes larger relative to the cost of increasing payments to creators on intensive markets, so coverage will increase in both equilibria. But the increase in t^* as a result of the subscription diminishes because with a flatter curve, more creators were already producing under the advertising-only model. Thus the intensive change in utility becomes more important. If the intensive change is initially so negative that the addition of the extensive markets cannot counterbalance it, then this disparity becomes more severe as the size of the markets becomes more even.

With the half normal distribution, consumer welfare increases as the market size function flattens. Again, because ads are already at the corner in the advertising-only model, there is no mechanism for consumer welfare to decrease with the introduction of the subscription, so consumers on the intensive markets are no worse off and flattening the market curvature very slightly *enhances* the increase in market coverage from the introduction of the subscription. more detailed results for this example are available in the online appendix.

4 Extensions

The baseline model is intentionally simple in to focus on the effects from adding the content creator side to the market, but the majority of its results are robust to relaxing these simplifying assumptions.

15. Although not immediately visible, ΔU is decreasing in α.

Heterogenous Utility

Allow the gross utility of viewing in a content market where the content creator is producing to vary along the interval $[\underline{u}, \bar{u}]$ with population density function $j(u)$ independent of $f(\cdot)$ and $g(\cdot)$.

[Figure 6 goes here]

Without the subscription, the critical level of η below which consumers choose to watch now depends on u. As u increases, consumers with increasingly high η find it worthwhile to watch. From Equation (1), the critical η varies linearly with u, as illustrated in Figure 6a[16].

The major changes occur when the platform can provide the subscription. Consumers whose utility is below the subscription price are choosing between not watching and the free version, whereas consumers whose utility is above it choose between the free and paid versions. Figure 6b shows the decisions of consumers as a function of their type when the subscription is available. Consumers whose utility of viewing is below $\tilde{p}_s$ behave as in the advertising-only version of the model. When they value viewing more than the price of the subscription they behave as in the baseline model with the subscription because watching will always be better than the outside option. Consumers with low nuisance cost watch content with ads, consumers with high utility and high nuisance cost watch with the subscription, and consumers with low utility and high nuisance cost do not watch at all. It follows that the proportion of consumers who watch (denoted here by ψ) is

$$\begin{cases} \hat{\psi} = \int_{\underline{u}}^{\bar{u}} F\left(\frac{u}{\hat{a}}\right) j(u)du & \text{advertising-only} \\ \tilde{\psi} = \underbrace{\int_{\underline{u}}^{\tilde{p}_s} F\left(\frac{u}{\tilde{a}}\right) j(u)du}_{\text{Ad viewing consumers}} + \underbrace{(1 - J(\tilde{p}_s))}_{\text{Subscribing consumers}} & \text{with subscription} \end{cases} \tag{27}$$

16. More complex utility functions or choice spaces such as allowing the consumers to choose how much time they spend watching content yield results that are qualitatively similar to those presented here.

As with earlier notation, $\hat{\psi}$ denotes the viewership proportion in the advertising-only model and $\tilde{\psi}$ the proportion in the model when the subscription is available. The presence of non-viewing consumers in the subscription model means that the percentage of consumers who watch is not guaranteed to rise with the addition of the subscription. If the subscription price and advertising level are sufficiently high, viewership may even decrease. The platform's profit under advertising-only is almost identical to that in the baseline, substituting $\hat{\psi}$ for $F\left(\frac{u}{\hat{a}}\right)$, however with the addition of the subscription it becomes

$$\begin{aligned}\pi_{plat} =& \Bigg[\underbrace{\left(\int_{\underline{u}}^{\tilde{p}_s} F\left(\frac{u}{\tilde{a}}\right) j(u)du + F\left(\frac{\tilde{p}_s}{\tilde{a}}\right)(1 - J(\tilde{p}_s)) \right) \tilde{a}(\tilde{p}_a - \tilde{w}_a)}_{\text{Advertising revenue}} \\ &+ \underbrace{\left(1 - F\left(\frac{\tilde{p}_s}{\tilde{a}}\right)\right)(1 - J(\tilde{p}_s))(\tilde{p}_s - \tilde{w}_s)}_{\text{Subscription revenue}} \Bigg] G(\tilde{t}^*)\end{aligned} \tag{28}$$

The platform's advertising revenue comes from low utility consumers and high utility consumers with a low nuisance cost. When it increases the advertising level, it will drive some of the low utility consumers away from the platform instead of onto the subscription. If it increases the price, then some of the high utility, high nuisance cost consumers will leave the platform rather than switching to the version with advertising. In both cases the increased tradeoff means that the platform will be able to extract less of the consumers' value than in the baseline model. Because Assumption 2 does not make sense with heterogeneous utility, I impose the following assumption instead:

Assumption 3. $z < \underline{u}$

Assumption 3 ensures that $\tilde{p}_s > \tilde{p}_a$. The platform set $p_a, a, w_a, \tilde{w}_s,$ and $\tilde{p}_s$ to maximize profit. Define

$$\tilde{w}' = \tilde{w}_a \tilde{a} \left(\int_{\underline{u}}^{\tilde{p}_s} j(u) F(\frac{u}{\tilde{a}} du + (1 - J(\tilde{p}_s)) F(\frac{\tilde{p}_s}{\tilde{a}}) \right) + \tilde{w}_s (1 - F(\frac{\tilde{p}_s}{\tilde{a}})) \tag{29}$$

and

$$\tilde{p}' = \tilde{p}_a\tilde{a}\left(\int_{\underline{u}}^{\tilde{p}_s} j(u)F(\frac{u}{\tilde{a}}du + (1-J(\tilde{p}_s))F(\frac{\tilde{p}_s}{\tilde{a}})\right) + \tilde{p}_s(1-F(\frac{\tilde{p}_s}{\tilde{a}})) \tag{30}$$

These are the analogues of $\tilde{p}$ and $\tilde{w}$ in the model with with heterogeneous gross utility of watching. The equilibria when the subscription is and is not available are characterized by Lemma 4

Lemma 4. *Under Assumption 1, Assumption 3 and if $g(\cdot)$ is concave, then*

1. *The price of advertising is $\hat{p}_a = z = \tilde{p}_a$.*

2. *In the advertising-only case, $\hat{w}$ solves Equation (9) substituting $\hat{\psi}$ for $F\left(\frac{u}{\hat{a}}\right)$. The advertising level $\hat{a}$ is the solution to*

$$\int_{\underline{u}}^{\bar{u}} \left(F\left(\frac{u}{\hat{a}}\right) - \frac{u}{\hat{a}}f\left(\frac{u}{\hat{a}}\right)\right) j(u)du = 0 \tag{31}$$

if $a \in (0,1]$ which solves this equation exists, otherwise $\hat{a} = 1$.

3. *When the platform can offer the subscription, $\tilde{a} \geq \hat{a}$ and the equilibrium advertising level is interior if there exists $\tilde{a} \in (0,1]$ solves the following equation:*

$$\begin{aligned}&\frac{\tilde{p}_s}{\tilde{a}^2}(1-J(\tilde{p}_s))(\tilde{p}_s - \tilde{a}\tilde{p}_a)f(\frac{\tilde{p}_s}{\tilde{a}}) + (\tilde{p}_a - \tilde{w}_a)F(\frac{\tilde{p}_s}{\tilde{a}})J(\tilde{p}_s)\\ =&\int_{\underline{u}}^{\tilde{p}_s}\left(\frac{u}{\tilde{a}}f(\frac{u}{\tilde{a}}) - F(\frac{u}{\tilde{a}})\right) j(u)du(\tilde{p}_a - \tilde{w}_a)\end{aligned} \tag{32}$$

   otherwise advertising will reach the corner solution at 1.

   The profit maximizing subscription price solves

$$1 = \frac{f(\frac{\tilde{p}_s}{a})}{1-F(\frac{\tilde{p}_s}{a})}\frac{\tilde{p}_s - \tilde{a}\tilde{p}_a}{a} + \frac{j(\tilde{p}_s)}{1-J(\tilde{p}_s)}(\tilde{p}_s - \tilde{w}_s) \tag{33}$$

if the solution is greater than $\underline{u}$. Otherwise $\tilde{p}_s = \underline{u}$. Lemma 2 continues to hold, substituting $\tilde{p}'$ and $\tilde{w}'$ for $\tilde{p}$ and $\tilde{w}$, so $\tilde{w}'$ is determined by Equation (17).

Equation (31), which determines the advertising level in the advertising-only model with heterogeneous utility, should be interpreted identically to the baseline equilibrium. The platform is balancing additional advertising against the lost viewership due to the increased nuisance cost, but it is now averaging the effects of the change in advertising across the utility space.

The terms on the left hand side of Equation (32) represent the net marginal benefit of increasing advertising from the consumers with $u \geq \tilde{p}_s$ because these consumers are either watching more ads or subscribing. The term on the right hand is the negative of the change in ad views from the consumers who do not find the subscription worthwhile. In equilibrium, the advertising level is above that in the advertising-only model even if the solution to Equation (32) is interior. Increasing the advertising level will cause some of the marginal consumers to switch to the subscription rather than leaving the platform. The tradeoff to increasing the advertising level, although not eliminated, is reduced.

The subscription pricing equation is similar to the one in the baseline model, but the platform faces an additional tradeoff in that some consumers will switch to not watching from purchasing the subscription rather than switching to ad-supported to version when the price increases. Because of this factor, the subscription price will be lower than the price in the baseline would be for the same distribution of nuisance costs.

Most of the results from Section 3 are robust to the addition of heterogeneous utility.

Proposition 2. *If Assumption 1, Assumption 3 and concavity of $g(\cdot)$ hold, then when consumers have heterogeneous utility*

1. *$\tilde{t}^* \geq \hat{t}^*$ and creators receive more revenue when the subscription is available than when the subscription is not available.*

2. *The welfare comparison between advertising-only and the equilibrium with the subscrip-*

tion for intensive market consumers who have $u \geq \tilde{p}_s$ will depend on their nuisance cost exactly as in points 2-4 of Proposition 1.

3. *Intensive market consumers for whom $u < \tilde{p}_s$ have weakly lower net utility with the presence of the subscription.*

4. *All extensive market consumers have weakly greater net utility when the subscription is available.*

Because the platform is free to set $\tilde{p}_s$ arbitrarily high and keep the advertising solution from the advertising-only model, platform profit, creator payment and content coverage must weakly increase with the addition of the subscription. Total welfare will increase under most distributions and parameter sets. The most significant difference comes in the form of comparative statics. With homogeneous utility, the more sensitive consumers are to advertising (i.e. the more mass is concentrated at high η), the more consumer surplus the platform can capture. With heterogeneous utility the optimal subscription price may be high enough that most consumers are still choosing between not watching and the ad-supported version of the service. But in this case a significant portion of the profit is still coming from advertising, therefore the platform may wish to set a lower advertising level as ad sensitivity increases, reducing the number of consumers purchasing the subscription.

The set of consumers whose welfare increases with the subscription is reduced compared to the baseline model. Consumers on the intensive markets are better off if they have a high nuisance cost *and* a high enough utility to purchase the subscription. All other intensive consumers are weakly worse off. The limitations which heterogeneous utility puts on the platform's ability to extract consumer value will lead to a smaller extensive effect, but the extensive effect will still be positive. Consumers on the extensive markets are gain from the subscription if they will watch the content, and those who do not watch are no worse off than when no content was produced.

Enriched Advertising Sector

The welfare results from the baseline model are also robust to an enriched advertising sector. Suppose we have a unit mass of advertisers with value of trade $z \in [\underline{z}, \bar{z}]$ which follows the log-concave population distribution $h(z)$. Then each advertiser will buy advertising if they have positive expected profit from doing so

$$z - p_a \geq 0 \tag{34}$$

For a given price, the demand for advertising will be $(1 - H(p_a))$. The platform can no longer set the advertising level independently of the advertising price. For the sake of readability and to provide an equivalent to a in the baseline model, I define the advertising level as $i(p_a) \equiv (1 - H(p_a))$. In the advertising-only model, platform profit is analogous to the baseline with no subscription

$$\pi_{plat} = (\hat{p}_a - \hat{w}_a) i(\hat{p}_a) G(\hat{t}^*) F(\frac{u}{i(\hat{p}_a)}) \tag{35}$$

Platform profit in the subscription model is similar to the baseline with the subscription.

$$\pi_{plat} = \left[(\tilde{p}_a - \tilde{w}_a) i(\tilde{p}_a) F\left(\frac{\tilde{p}_s}{i(\tilde{p}_a)}\right) + (\tilde{p}_s - \tilde{w}_s)(1 - F\left(\frac{\tilde{p}_s}{i(\tilde{p}_a)}\right)) \right] G(\tilde{t}^*) \tag{36}$$

Consumers behave as in the baseline model with the subscription replacing a with $i(p_a)$. The enriched advertising sector does not directly change the behavior of the consumers or content creators from the baseline model except insofar as it changes the advertising level. Before solving for equilibrium I impose the following assumption:

Assumption 4. *In the model with heterogenous advertisers*

- $\bar{z} < u$
- $\left(\frac{x}{1-H(x)}\right)^2 \frac{f\left(\frac{x}{1-H(x)}\right)}{F\left(\frac{x}{1-H(x)}\right)}$ *is increasing in* x

The first part of Assumption 4 ensures that the price of the subscription is greater than the price of advertising. As in Section 4, I focus on equilibria with this property because it is most relevant to the motivating examples. The second part of the assumption ensures that the platform's profit is quasi-concave in the advertising price when it can offer the subscription. Note that Assumption 1 requires that $\left(\frac{x}{1-H(x)}\right) \frac{f\left(\frac{x}{1-H(x)}\right)}{F\left(\frac{x}{1-H(x)}\right)}$ is *decreasing* in x, so the two assumptions together place upper and lower bounds on the log-concavity of $f(\cdot)$ and a lower bound on $\underline{\eta}$.

Lemma 5. *Under Assumption 1, Assumption 4 and if* $g(\cdot)$ *is concave, then*

1. $\hat{w}$ *is determined by Equation* (9) *and* $\tilde{w}$ *by Equation* (17)*, substituting* a *with* $i(p_a)$ *in both equations. Similarly, the subscription price is determined by Equation* (19) *using the same substitution.*

2. *With advertising only the price of advertising solves*

$$\frac{1-H(\hat{p}_a)}{h(\hat{p}_a)} F(\frac{u}{i(\hat{p}_a)}) = \left(F(\frac{u}{i(\hat{p}_a)}) - f(\frac{u}{i(\hat{p}_a)})\frac{u}{i(\hat{p}_a)}\right)\hat{p}_a \tag{37}$$

   if a solution such that $\bar{z} \geq \hat{p}_a \geq \underline{z}$ *exists. Otherwise it will be at the corner where* $i(\hat{p}_a) = 1$ *and* $\hat{p}_a = \underline{z}$*.*

3. *In the model with the subscription,* $\tilde{p}_a \leq \hat{p}_a$ *and the advertising price solves*

$$\frac{1-H(\tilde{p}_a)}{h(\tilde{p}_a)} F(\frac{\tilde{p}_s}{i(\tilde{p}_a)}) = \left(\frac{\tilde{p}_s}{i(\tilde{p}_a)}\right)^2 f(\frac{\tilde{p}_s}{i(\tilde{p}_a)}) + p_a \left(F(\frac{\tilde{p}_s}{i(\tilde{p}_a)}) - \frac{\tilde{p}_s}{i(\tilde{p}_a)} f(\frac{\tilde{p}_s}{i(\tilde{p}_a)})\right) \tag{38}$$

if a solution such that $\bar{z} \geq \tilde{p}_a \geq \underline{z}$ exists. Otherwise it will be at the corner where $i(\tilde{p}_a) = 1$ and $\tilde{p}_a = \underline{z}$.

Ad views are maximized when $F(\frac{u}{i(p_a)}) = f(\frac{u}{i(p_a)})\frac{u}{i(p_a)}$, but when ad views are maximized the marginal profit from increasing the ad price is strictly positive. Therefore in equilibrium the number of total ad views must be below that in the baseline model so that the right hand side of Equation (37) is positive. Note that the platform is no longer maximizing advertising views so the platform's incentives and the creators' objective are no longer perfectly aligned. The creators will always prefer more advertising than the platform is willing to provide.

Because the platform is not directly controlling the advertising level, reducing the price may have such a minute effect on advertising that the additional subscriptions it would drive do not make up for the lost advertising revenue. In this case, the intensive market consumers are benefiting from price insensitivity of advertisers, but this benefit comes at the cost of reduced welfare gains for the extensive markets. This limit on the advertising level will mitigate the increase in content coverage and some consumers will not have content produced for them when they would if the advertising level were higher.

Proposition 3. *If Assumption 1, Assumption 4 and concavity of $g(\cdot)$ hold, then in the model with the enriched advertising sector*

1. *Platform profits, creator payments and content coverage are all higher when the platform can offer the subscription.*

2. *Consumers' welfare changes with the presence of the subscription in the same manner as Proposition 1, replacing $\hat{a}$ with $i(\hat{p}_a)$.*

3. *Advertiser profits can increase or decrease as a result of the subscription.*

The ambiguity of the subscription's effect on advertiser profits is surprising. Intuitively, the advertisers are losing ad views on the intensive market because some consumers will be using the premium version of the content rather than viewing ads. However, because the platform wishes to increase the advertising level, it will lower the advertising price, meaning that the expected profit for each ad view increases. Additionally, content on the extensive markets brings in new ad-viewing consumers to the platform which mitigates the loss of the ad-viewing consumers on the intensive markets. The total ad views for each advertiser may even increase.

5 Recommendation Algorithms

The assumption that there is little audience crossover is a significant boon for tractability, but it is significantly less true for platforms like YouTube than it is for livestreaming services such as Twitch.tv. The way that viewers are matched with creators other than the one which attracted them to the platform in the first place is largely a result of recommendations by the platform. As Dinerstein et al. (2018) demonstrate, the algorithm which a platform uses to suggest content to users can have a significant impact on the behavior of platform participants. While I do not explicitly model the search process out of a desire to keep this article at a manageable length, my model can provide a framework for considering its impact.

With advertising as the only revenue source, the platform's profit depends on the number of ad views, so there is little revenue difference if a viewer watches a video through to the end or not since most advertising comes at the beginning of the video. In fact it is often better for the platform if the viewer does not finish watching a video and instead moves on to a new video since this increases the number of advertisements to which they are exposed. As a result the platform designs its recommendation algorithm to maximize the number of videos a viewer watches while still remaining with the platform. As the consumer search process for media often involves watching the first part of videos being considered, this effect implies that the platform will design its video recommendation algorithm to maximize the number of times consumers search.

On the other hand, with a subscription the number of videos watched by each viewer has less impact than how much they value the content they consume. As demonstrated in Section 4, the more consumers value content, the more the platform can charge for the subscription. In this case, the platform will design its recommendation algorithm to maximize the surplus of the searcher rather than the number of searches. If the platform's information about video quality is noisy, this leads to an emphasis on content which already has many views (since a high view count indicates high quality) and can lead to fewer consumers being exposed to niche content[17]. While creators' revenue per viewer increases, this change in the content recommendation algorithm can lead to a reduction in the number of viewers for niche content. This does not necessarily mean that the extensive market effect disappears, but instead that creators with smaller audience sizes will be discouraged from experimenting with content outside of the videos which attract their core audience.

This pattern also explains why Twitch.tv places much more emphasis on subscriptions than YouTube. Because Twitch is a livestreaming service, viewers expect to watch a single piece of content for an extended period. When channels do take commercial breaks they tend to be more sporadic than on YouTube. The ability to interact with streamers in a live setting also creates a higher potential for engagement (and therefore a higher value for the content) than with the relatively short and non-interactive video on YouTube.

6 Conclusion

I explore a model where a media platform must attract content creators in addition to consumers and advertisers, and evaluate the welfare effects of a subscription which allows consumers to view content without exposure to advertising. I find that the provision of an ad-avoidance subscription can increase aggregate consumer welfare and will generally increase total welfare. Tåg's (2009) result that consumer welfare decreases holds if either the consumers are too sensitive to advertising so that the subscription price is high, or too

17. I thank an anonymous referee for suggesting relative emphasis of niche content as an avenue of consideration.

insensitive so that few consumers take advantage of the subscription. On the other hand, if consumers' advertising sensitivity lies in between these two extremes then the subscription will increase consumer welfare. The more sensitive consumers are to advertising, the more the benefits of the subscription will be captured by the platform and content producers. This capture creates a tradeoff between the welfare of consumers on the intensive markets and those on the extensive markets. As platform profits increase, so too do payments to creators and therefore the number of creators who produce. Because the extensive market consumers are always at least weakly better off when their content producer is active, the welfare effect from the addition of this content will be strictly positive. Although it may be negligible if the platform is able to capture too much consumer welfare.

The intuitive conclusion is that introducing the subscription would reduce the profits of advertisers by reducing their potential audience, under many parameter specifications, the opposite holds true. I show in Section 4 that the platform has an incentive to increase the amount of advertising in order to drive more consumers on to the subscription, which lowers the price of purchasing ad views. Additionally, the increase in content coverage means that the number of total ad views may increase rather than decrease. Consequently the advertisers may be *better off* with the presence of a subscription than without it.

There are a number of other potential implications from separation of content from the platform which are not covered in this model. In a monopoly, the platform would prefer to pay the creators on the largest markets less per view and smaller creators more in order to increase the amount of content and bring in more consumers. If the platform is competing with another platform, then the largest creators become the most valuable way of attracting consumers. The ability to differentiate might increase competition for these creators and *increase* the payments they receive[18]. Additionally, in this model most of the benefit of the subscription to content creators is accrued by the large creators who have a significant profit margin over their cost of production. From discussions with a content creator who makes

18. Assuming that creators, following the terminology of Armstrong (2006), are single homing.

a living on YouTube, the most significant benefit of the subscription is to smaller creators. This is because most YouTube channels face a dynamic problem where they have to decide how much time and money to invest in improving their content, and the revenue they earn from advertising is highly variable and unpredictable. The subscription not only increases the revenue these creators earn, but is also significantly more reliable, thus reducing their uncertainty when deciding how much to invest in their channel.

References

Alexandrov, Alexei, and Joyee Deb. 2012. Price discrimination and investment incentives. *International Journal of Industrial Organization* 30 (6): 615–623.

Anderson, Simon P., and Stephen Coate. 2005. Market provision of broadcasting: a welfare analysis. *The Review of Economic Studies* 72 (4): 947–972. ISSN: 00346527, 1467937X. http://www.jstor.org/stable/3700696.

Anderson, Simon P., and Joshua S. Gans. 2011. Platform siphoning: ad-avoidance and media content. *American Economic Journal: Microeconomics* 3, no. 4 (): 1–34. doi:10.1257/mic.3.4.1. http://www.aeaweb.org/articles?id=10.1257/mic.3.4.1.

Armstrong, Mark. 2006. Competition in two-sided markets. *The RAND Journal of Economics* 37 (3): 668–691.

Bergemann, Dirk, and Alessandro Bonatti. 2011. Targeting in advertising markets: implications for offline versus online media. *The RAND Journal of Economics* 42 (3): 417–443. ISSN: 07416261. http://www.jstor.org/stable/23046807.

Choi, Jay Pil. 2006. Broadcast competition and advertising with free entry: subscription vs. free-to-air. *Information Economics and Policy* 18 (2): 181–196. ISSN: 0167-6245. doi:https://doi.org/10.1016/j.infoecopol.2005.12.002. http://www.sciencedirect.com/science/article/pii/S0167624506000047.

Crampes, Claude, Carole Haritchabalet, and Bruno Jullien. 2009. Advertising, competition and entry in media industries. *The Journal of Industrial Economics* 57 (1): 7–31. ISSN: 1467-6451. doi:10.1111/j.1467-6451.2009.00368.x. http://dx.doi.org/10.1111/j.1467-6451.2009.00368.x.

Dinerstein, Michael, Liran Einav, Jonathan Levin, and Neel Sundaresan. 2018. Consumer price search and platform design in internet commerce. *American Economic Review* 108 (7): 1820–59.

Green, Hank. 2015. The $1,000 cpm. Accessed January 16, 2017. https://medium.com/@hankgreen/the-1-000-cpm-f92717506a4b#.3lxhobg3q.

Gutelle, Sam. 2014. The average youtube cpm is $7.60, but making money isn't easy. Accessed April 24, 2018. https://www.tubefilter.com/2014/02/03/youtube-average-cpm-advertising-rate/.

Hagiu, Andrei, and Bruno Jullien. 2011. Why do intermediaries divert search? *The RAND Journal of Economics* 42 (2): 337–362.

Hagiu, Andrei, and Daniel Spulber. 2013. First-party content and coordination in two-sided markets. *Management Science* 59 (4): 933–949.

Hagiu, Andrei, and Julian Wright. 2015. Multi-sided platforms. *International Journal of Industrial Organization* 43:162–174.

Maldonado, Jose Francisco Tudon. 2017. Congestion v content provision in net neutrality: the case of amazon's twitch.tv. Working paper.

Peitz, Martin, and Tommaso M Valletti. 2008. Content and advertising in the media: pay-tv versus free-to-air. *International Journal of industrial organization* 26 (4): 949–965.

Prasad, Ashutosh, Vijay Mahajan, and Bart Bronnenberg. 2003. Advertising versus pay-per-view in electronic media. *International Journal of Research in Marketing* 20, no. 1 (): 13–30. ISSN: 0167-8116. doi:10.1016/S0167-8116(02)00119-2.

Tåg, Joacim. 2009. Paying to remove advertisements. *Information Economics and Policy* 21 (4): 245–252. ISSN: 0167-6245. doi:http://dx.doi.org/10.1016/j.infoecopol.2009.02.001. http://www.sciencedirect.com/science/article/pii/S0167624509000377.

White, Alexander. 2013. Search engines: left side quality versus right side profits. *International Journal of Industrial Organization* 31 (6): 690–701.

Wilbur, Kenneth C. 2008. A two-sided, empirical model of television advertising and viewing markets. *Marketing Science* 27 (3): 356–378. doi:10.1287/mksc.1070.0303. eprint: http://dx.doi.org/10.1287/mksc.1070.0303. http://dx.doi.org/10.1287/mksc.1070.0303.

Yang, Huanxing. 2013. Targeted search and the long tail effect. *RAND Journal of Economics* 44 (4): 733–756. http://EconPapers.repec.org/RePEc:bla:randje:v:44:y:2013:i:4:p:733-756.

A Proofs

Proof of Lemma 1

Begin by noting the following identities:

$$\frac{\partial t^*}{\partial w_a} = \frac{1}{g'(t^*)} \frac{-c}{{w_a}^2 a F(\frac{u}{a})} > 0 \tag{39}$$

$$\frac{\partial t^*}{\partial a} = \frac{-c}{g'(t^*)} \frac{w_a F(\frac{u}{a}) + a w_a \frac{\partial F(\frac{u}{a})}{\partial a}}{({w_a}^2 a F(\frac{u}{a}))^2} > 0 \tag{40}$$

and

$$\frac{\partial F(\frac{u}{a})}{\partial a} = -f(\frac{u}{a})\frac{u}{a^2} \tag{41}$$

Substituting Equation (39) and Equation (41) into Equation (40) gives

$$\frac{\partial t^*}{\partial a} = \frac{-cw_a}{g^{'}(t^*)}\frac{F(\frac{u}{a}) - f(\frac{u}{a})\frac{u}{a}}{({w_a}^2 aF(\frac{u}{a}))^2} > 0 \tag{42}$$

The first term in the product is positive as $g^{'}(t) < 0$. Thus the sign of the derivative is positive iff the increased revenue from having more ad views ($F(\frac{u}{a})$) outweighs the loss in viewership caused by the increase in nuisance ($f(\frac{u}{a})\frac{u}{a}$).

Take the FOC with regard to advertising level:

$$\begin{aligned}\frac{\partial \pi_{plat}}{\partial a} =&(p_a - w_a)\left[G(t^*)F(\frac{u}{a}) + g(t^*)\frac{\partial t^*}{\partial a}aF(\frac{u}{a}) + aG(t^*)\frac{\partial F(\frac{u}{a})}{\partial a}\right] \\ =&(p_a - w_a)\left[G(t^*)\left(F(\frac{u}{a}) - f(\frac{u}{a})\frac{u}{a}\right) + g(t^*)\frac{\partial t^*}{\partial a}aF(\frac{u}{a})\right]\end{aligned} \tag{43}$$

Using the identities established above and the definition of t^*, then pulling out common factors:

$$\frac{\partial \pi_{plat}}{\partial a} = (p_a - w_a)\left(F(\frac{u}{a}) - f(\frac{u}{a})\frac{u}{a}\right)\left[G(t^*) - \frac{aF(\frac{u}{a})}{g^{'}(t^*)}\frac{c^2}{(w_a aF(\frac{u}{a}))^3}\right] \tag{44}$$

Because $g(\cdot)$ is decreasing all terms in this product are positive with the exception of $\left(F(\frac{u}{a}) - f(\frac{u}{a})\frac{u}{a}\right)$, so this is the only relevant part of the platform's decision. So long as Assumption 1 is satisfied, there will be a unique solution. This solution must be profit maximizing because (from Assumption 1 and the fact that $f(\cdot)$ is log concave) $\left(F(\frac{u}{a}) - f(\frac{u}{a})\frac{u}{a}\right)$

is positive for a close to 0 and negative for a close to 1.

Now take the derivative of platform profit with regard to creator payment, substituting appropriately using the identities above:

$$\frac{\partial \pi_{plat}}{\partial w_a} = aF(\frac{u}{a}) \left(\frac{g(t^*)}{g^{'}(t^*)} \frac{-c}{{w_a}^2 aF(\frac{u}{a})} (p_a - w_a) - G(t^*) \right) \tag{45}$$

Using the definition of t^*

$$\frac{\partial \pi_{plat}}{\partial w_a} = aF(\frac{u}{a}) \left(\frac{1}{g^{'}(t^*)} \frac{-c^2}{w_a (w_a aF(\frac{u}{a}))^2} (p_a - w_a) - G(t^*) \right) \tag{46}$$

The derivative will be positive when $G(t^*) = 0$ and negative at $w_a = p_a$, it is also continuous, so a solution always exists. Checking for concavity, take the second derivative

$$\begin{aligned} \frac{\partial^2 \pi_{plat}}{\partial^2 w_a} = aF(\frac{u}{a}) \Bigg(& \frac{-1}{g^{''}(t^*)} \frac{\partial t^*}{\partial w_a} \frac{-c^2}{w_a (w_a aF(\frac{u}{a}))^2} (p_a - w_a) \\ & - \frac{3}{g^{'}(t^*)} \frac{-c^2}{{w_a}^4 (aF(\frac{u}{a}))^2} (p_a - w_a) \\ & - \frac{1}{g^{'}(t^*)} \frac{-c^2}{w_a (w_a aF(\frac{u}{a}))^2} - g(t^*) \frac{\partial t^*}{\partial w_a} \Bigg) \end{aligned} \tag{47}$$

So long as the first term is negative and/or sufficiently small, the platform's profit is concave in prices. Thus $g^{''}(t) \leq 0$ is sufficient, but not necessary to ensure that there is a unique profit maximizing creator payment for every advertising level so long as there are a positive number of views. ■

Proof of Lemma 2

The following identities are useful:

$$\frac{\partial t^*}{\partial \tilde{w}_s} = \frac{-c}{g^{'}(t^*)} \frac{1 - F(\frac{\tilde{p}_s}{\tilde{a}})}{\left[\tilde{w}_a \tilde{a} F(\frac{\tilde{p}_s}{\tilde{a}}) + \tilde{w}_s (1 - F(\frac{\tilde{p}_s}{\tilde{a}}))\right]^2} > 0 \tag{48}$$

$$\frac{\partial t^*}{\partial \tilde{w}_a} = \frac{-c}{g'(t^*)} \frac{aF(\frac{\tilde{p}_s}{\tilde{a}})}{\left[\tilde{w}_a \tilde{a} F(\frac{\tilde{p}_s}{\tilde{a}}) + \tilde{w}_s(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))\right]^2} > 0 \tag{49}$$

Using the above identities, take the derivative of platform profit with regard to the share of subscription fees ($\tilde{w}_s$)

$$\begin{aligned} \frac{\partial \pi_{plat}}{\partial \tilde{w}_s} = & \left[(p_a - \tilde{w}_a)\tilde{a}F(\frac{\tilde{p}_s}{\tilde{a}}) + (\tilde{p}_s - \tilde{w}_s)(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))\right] g(t^*) \frac{\partial t^*}{\partial \tilde{w}_s} \\ & - (1 - F(\frac{\tilde{p}_s}{\tilde{a}}))G(t^*) \end{aligned} \tag{50}$$

and advertising revenue ($\tilde{w}_a$)

$$\begin{aligned} \frac{\partial \pi_{plat}}{\partial \tilde{w}_a} = & \left[(p_a - \tilde{w}_a)\tilde{a}F(\frac{\tilde{p}_s}{\tilde{a}}) + (\tilde{p}_s - \tilde{w}_s)(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))\right] g(t^*) \frac{\partial t^*}{\partial \tilde{w}_a} \\ & - \tilde{a}F(\frac{\tilde{p}_s}{\tilde{a}})G(t^*) \end{aligned} \tag{51}$$

Both of these derivatives give the same FOC.

$$\frac{-c^2}{g'(t^*)} \frac{(p_a - \tilde{w}_a)aF(\frac{\tilde{p}_s}{\tilde{a}}) + (\tilde{p}_s - \tilde{w}_s)(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))}{\left[\tilde{w}_a a F(\frac{\tilde{p}_s}{\tilde{a}}) + \tilde{w}_s(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))\right]^3} = G(t^*) \tag{52}$$

For any pair $(\tilde{w}_a, \tilde{w}_s)$ that solves this FOC, $(\tilde{w}_a + \epsilon, \tilde{w}_s - \frac{aF(\frac{\tilde{p}_s}{\tilde{a}})}{(1-F(\frac{\tilde{p}_s}{\tilde{a}}))}\epsilon)$ will also be a solution for any $\epsilon \in \mathbb{R}$, thus there is a continuum of solutions. From the definition of $\tilde{p}$ and $\tilde{w}$, I can rewrite this FOC as

$$\frac{-c^2}{g'(\tilde{t}^*)} \frac{\tilde{p} - \tilde{w}}{\tilde{w}^3} = G(\tilde{t}^*) \tag{53}$$

By almost the exact same derivation as in the proof of Lemma 1, concavity of $g(\cdot)$ gives uniqueness and sufficiency of this FOC for profit maximization.

■

Proof of Lemma 3

The value of $\tilde{p}_a$ was derived in text. The value of $\tilde{w}$ comes from Lemma 2. To see the increase in advertising level, take the derivative of platform profit with regard to advertising level

$$\begin{aligned}\frac{\partial \pi_{plat}}{\partial a} = \Bigg[(\tilde{p}_a - \tilde{w}_a)\left(F(\frac{\tilde{p}_s}{\tilde{a}}) - \tilde{a} f(\frac{\tilde{p}_s}{\tilde{a}})\frac{\tilde{p}_s}{\tilde{a}^2}\right) \\ + (\tilde{p}_s - \tilde{w}_s) f(\frac{\tilde{p}_s}{\tilde{a}})\frac{\tilde{p}_s}{\tilde{a}^2}\Bigg] G(\tilde{t}^*) \\ + (\tilde{p} - \tilde{w}) g(\tilde{t}^*)\frac{\partial \tilde{t}^*}{\partial a}\end{aligned} \tag{54}$$

Using Equation (13) and taking the derivative of $\tilde{t}^*$ w.r.t. a

$$\begin{aligned}\frac{\partial \pi_{plat}}{\partial a} = \Bigg[(\tilde{p}_a - \tilde{w}_a)\left(F(\frac{\tilde{p}_s}{\tilde{a}}) - \tilde{a} f(\frac{\tilde{p}_s}{\tilde{a}})\frac{\tilde{p}_s}{\tilde{a}^2}\right) \\ + (\tilde{p}_s - \tilde{w}_s) f(\frac{\tilde{p}_s}{\tilde{a}})\frac{\tilde{p}_s}{\tilde{a}^2}\Bigg] G(\tilde{t}^*) \\ + (\tilde{p} - \tilde{w})\frac{-c^2}{g'(\tilde{t}^*)}\frac{\tilde{w}_a F(\frac{\tilde{p}_s}{\tilde{a}}) + \frac{\tilde{p}_s}{\tilde{a}^2}(\tilde{w}_s - \tilde{a}\tilde{w}_a)}{\left[\tilde{w}_a \tilde{a} F(\frac{\tilde{p}_s}{\tilde{a}}) + \tilde{w}_s(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))\right]^3}\end{aligned} \tag{55}$$

Using Lemma 2, I can set $\tilde{a}\tilde{w}_a = \tilde{w}_s$. Then from Equation (52)

$$\begin{aligned}\frac{\partial \pi_{plat}}{\partial a} &= \left[(\tilde{p}_a - \tilde{w}_a) F(\frac{\tilde{p}_s}{\tilde{a}}) + (\tilde{p}_s - a\tilde{p}_a)\frac{\tilde{p}_s}{a^2} f(\frac{\tilde{p}_s}{\tilde{a}}) + \tilde{w}_a F(\frac{\tilde{p}_s}{\tilde{a}})\right] G(t^*) \\ \Rightarrow \frac{\partial \pi_{plat}}{\partial a} &= \left[\tilde{p}_a F(\frac{\tilde{p}_s}{\tilde{a}}) + (\tilde{p}_s - a\tilde{p}_a)\frac{\tilde{p}_s}{a^2} f(\frac{\tilde{p}_s}{\tilde{a}})\right] G(t^*)\end{aligned} \tag{56}$$

This derivative is unambiguously positive if $\tilde{p}_s > \tilde{a}z = \tilde{a}\tilde{p}_a$, so if that inequality holds advertisement will to a corner solution at 1. $\tilde{p}_s < \tilde{a}z$ only if $\tilde{p}_s = u < \tilde{a}z$ (see below) then we can rearrange the above to get

$$\frac{\partial \pi_{plat}}{\partial a} = \left[\tilde{p}_a \left(F(\frac{u}{\tilde{a}}) - \frac{u}{a} f(\frac{u}{\tilde{a}})\right) + \left(\frac{u}{\tilde{a}}\right)^2 f(\frac{u}{\tilde{a}})\right] G(t^*) \tag{57}$$

$\hat{a}$ sets $F(\frac{u}{\tilde{a}}) - \frac{u}{a} f(\frac{u}{\tilde{a}}) = 0$, so this derivative will be positive at $\tilde{a} = \hat{a}$, so the profit maximizing advertising level must be above $\hat{a}$. Furthermore, Assumption 1 and log concavity of $f(\cdot)$ will be enough to give sufficiency of this equation for profit maximization.

To find the subscription fee, take the first order derivative of platform profit

$$\frac{\partial \pi_{plat}}{\partial \tilde{p}_s} = \left[(\tilde{p}_a - w_a)\tilde{a} f(\frac{\tilde{p}_s}{\tilde{a}})\frac{1}{\tilde{a}} + (1 - F(\frac{\tilde{p}_s}{\tilde{a}})) - (\tilde{p}_s - \tilde{w}_s) f(\frac{\tilde{p}_s}{\tilde{a}})\frac{1}{\tilde{a}}\right] G(t^*) + (\tilde{p} - \tilde{w}) g(t^*) \frac{\partial t^*}{\partial \tilde{p}_s} \tag{58}$$

Using Equation (13) and taking the derivative of $\tilde{t}^*$ w.r.t. $\tilde{p}_s$ with $\tilde{a}\tilde{w}_a = \tilde{w}_s$

$$\frac{\partial \pi_{plat}}{\partial \tilde{p}_s} = G(t^*) \left[(1 - F(\frac{\tilde{p}_s}{\tilde{a}})) + (\tilde{p}_a \tilde{a} - \tilde{p}_s) f(\frac{\tilde{p}_s}{\tilde{a}})\frac{1}{a}\right] \tag{59}$$

If $z\tilde{a} = \tilde{p}_a \tilde{a} > \tilde{p}_s$ then this derivative will be strictly positive, so the only circumstance in which the platform will set the subscription price less than $\tilde{a}z$ is when $\tilde{a}z > u$ and the subscription price is $\tilde{p}_s = u$. If that inequality does not hold, the derivative above gives gives a relatively straightforward FOC

$$\frac{1 - F(\frac{\tilde{p}_s}{\tilde{a}})}{f(\frac{\tilde{p}_s}{\tilde{a}})} = \frac{\tilde{p}_s - \tilde{p}_a \tilde{a}}{\tilde{a}} \tag{60}$$

If $z > u$, then the platform will always set $\tilde{p}_s = u$ because for any advertising level in which the subscription price is greater than $\tilde{a}z$, the platform will want to increase the advertising level. But then as $\tilde{a}$ increases $\tilde{a}z$ will surpass $\tilde{p}_s$, and the platform will want to increase the subscription price. For $z > u$, the subscription price will reach the corner solution where it can no longer be increased after $\tilde{a}z$ increases. For $u > z$, the advertising level will reach the corner first and the pricing FOC becomes

$$\frac{1 - F(\tilde{p}_s)}{f(\tilde{p}_s)} = \tilde{p}_s - \tilde{p}_a \tag{61}$$

Profit is increasing in price if the inverse hazard function on the left hand side is greater than the margin on subscribing consumers on the right. It follows from log concavity of $f(\cdot)$ that the inverse hazard rate will be decreasing in the subscription price whereas the margin is increasing so the FOC can be satisfied at most once. For $p_s < \tilde{p}_s$ profit is increasing in p_s, whereas it will be decreasing in p_s if the opposite inequality holds. Therefore this FOC will be sufficient for profit maximization.■

Proof of Proposition 1

To show that the payment to creators will increase, consider Equation (53) from Lemma 2. The same first order condition can be used in the advertising-only equilibrium substituting $\hat{w} = \hat{w}_a \hat{a} F(\frac{u}{\hat{a}})$ for $\tilde{w}$ and $\hat{p} = \hat{p}_a \hat{a} F(\frac{u}{\hat{a}})$ for $\tilde{p}$ Because the platform solves this problem in terms of the respective w and p for both the advertising-only model and the model with the subscription. It is trivial to show that $\tilde{p} > \hat{p}$, and given that

$$t^* = g^{-1}\left(\frac{c}{w}\right) \tag{62}$$

in both problems then the solution to Equation (53) can be thought of as an implicit function of p, and we can see that the left hand side increases with p, meaning that the right hand side must as well. Thus $G(t^*)$ must increase with the subscription, meaning that both creator payments and variety of content will increase.

If consumers do not purchase the subscription when it is available, then they face more nuisance cost than they would by watching in the advertising-only equlibrium due to the higher advertising level from Lemma 3. Therefore they would have watched in that equilibrium, and they would have received greater net utility from doing so.

Consumers are better off in the equilibrium with the subscription only if they purchase the subscription and the price of the subscription is less than the nuisance cost they would face in the advertising-only equilibrium. i.e. if

$$\begin{aligned} \hat{a}\eta &> \tilde{p}_s \\ \implies \eta &> \frac{\tilde{p}_s}{\hat{a}} \end{aligned} \tag{63}$$

Consumers will purchase the subscription if $\eta > \tilde{p}_s$, and $\hat{a} \leq 1$, so unless $\hat{a} = 1$ $\frac{\tilde{p}_s}{\hat{a}} > \tilde{p}_s$, and there is a set of consumers who purchase the subscription, but the price of the subscription is greater than the nuisance cost they would have faced in the advertising-only equilibrium.

The extensive market consumers must be at least weakly better off from the presence of the subscription because they all choose to watch when their relevant content creator produces. The outside option is still available, so they can be no worse off than in the advertising-only equilibrium.■

Proof of Lemma 4

As before, a consumer deciding between the free version and not watching will watch iff

$$u \geq a\eta \tag{64}$$

which means that for a given value of u, only those consumers for whom $\eta \leq \frac{u}{a}$ will watch. There will be a density $j(u)$ of consumers with any given u, and the independence of the two population distributions leads to Equation (27). To get Equation (31) note that Equation (44) can be generalized to

$$\frac{\partial \hat{\pi}_{plat}}{\partial \hat{a}} = (\hat{p}_a - \hat{w}_a) \left(\hat{\psi} - \hat{a} \frac{\partial \hat{\psi}}{\partial \hat{a}} \right) \left[G(\hat{t}^*) + \hat{a} g(\hat{t}^*) \hat{\psi} \right] \tag{65}$$

with the middle term being the only one of ambiguous sign. In the case of heterogeneous utility $\hat{\psi} - \hat{a}\frac{\partial \hat{\psi}}{\partial \hat{a}} = \int_{\underline{u}}^{\bar{u}} \left(F\left(\frac{u}{\hat{a}}\right) - \frac{u}{\hat{a}} f\left(\frac{u}{\hat{a}}\right) \right) j(u) du$. Assumption 1 ensures that this derivative will have only one solution because it means that the middle term will be decreasing in a.

Equation (32) is simply derived by taking the derivative of profit with respect to $\tilde{a}$, and setting it equal to 0

$$\begin{aligned} \frac{\partial \tilde{\pi}_{plat}}{\partial \tilde{a}} =& \frac{\tilde{p}_s}{\tilde{a}^2} (1 - J(\tilde{p}_s))(\tilde{p}_s - \tilde{a}\tilde{p}_a) f(\frac{\tilde{p}_s}{\tilde{a}}) + (\tilde{p}_a - \tilde{w}_a) F(\frac{\tilde{p}_s}{\tilde{a}}) J(\tilde{p}_s) \\ &+ \int_{\underline{u}}^{\tilde{p}_s} \left(F(\frac{u}{\tilde{a}}) - \frac{u}{\tilde{a}} f(\frac{u}{\tilde{a}}) \right) j(u) du (\tilde{p}_a - \tilde{w}_a) \end{aligned} \tag{66}$$

Given that $\hat{a}$ sets $\int_{\underline{u}}^{\tilde{p}_s} \left(F(\frac{u}{\tilde{a}}) - \frac{u}{\tilde{a}} f(\frac{u}{\tilde{a}}) \right) j(u) du = 0$, the derivative above will be strictly positive at $\hat{a}$, and it is easy to show that the second derivative is negative given Assumption 1 and Assumption 3. The solution to Equation (32) is sufficient for profit maximization and $\tilde{a}$ must be greater than $\hat{a}$.

Equation (33) is derived by taking the derivative of the profit function with respect to $\tilde{p}_s$:

$$\begin{aligned} \frac{\partial \pi_{plat}}{\partial \tilde{p}_s} =& \Bigg[\left(F(\frac{\tilde{p}_s}{\tilde{a}}) j(\tilde{p}_s) + \frac{1}{\tilde{a}} f(\frac{\tilde{p}_s}{\tilde{a}})(1 - J(\tilde{p}_s)) - F(\frac{\tilde{p}_s}{\tilde{a}}) j(\tilde{p}_s) \right) \tilde{a}(\tilde{p}_a - \tilde{w}_a) \\ &- \frac{1}{\tilde{a}} f(\frac{\tilde{p}_s}{\tilde{a}})(1 - J(\tilde{p}_s)) - j(\tilde{p}_s)(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))(\tilde{p}_s - \tilde{w}_s) + (1 - F(\frac{\tilde{p}_s}{\tilde{a}}))(1 - J(\tilde{p}_s)) \Bigg] G(\tilde{t}^*) \end{aligned} \tag{67}$$

Focusing on the inside of the parentheses and setting the equation equal to 0

$$\begin{aligned} &(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))(1 - J(\tilde{p}_s)) \\ &= \frac{1}{\tilde{a}} f(\frac{\tilde{p}_s}{\tilde{a}})(1 - J(\tilde{p}_s))(\tilde{p}_s - \tilde{a}\tilde{p}_a) + j(\tilde{p}_s)(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))(\tilde{p}_s - \tilde{w}_s) \end{aligned} \tag{68}$$

Divide through by $(1 - F(\frac{\tilde{p}_s}{\tilde{a}}))(1 - J(\tilde{p}_s))$

$$1 = \frac{f(\frac{\tilde{p}_s}{\tilde{a}})}{1 - F(\frac{\tilde{p}_s}{\tilde{a}})} \frac{\tilde{p}_s - \tilde{a}\tilde{p}_a}{\tilde{a}} + \frac{j(\tilde{p}_s)}{1 - J(\tilde{p}_s)}(\tilde{p}_s - \tilde{w}_s) \tag{69}$$

Profit is increasing in $\tilde{p}_s$ if the left hand side is greater than the right, and decreasing if the opposite holds true. From log-concavity of $f(\cdot)$ and $j(\cdot)$ the right hand side is increasing in p_s whereas the left hand side is constant, meaning that the first order condition, if it can be satisfied, will be sufficient for profit maximization. If this condition cannot be satisfied then the price will be a corner at $\underline{u}$ because the right hand side approaches infinity as p_s approaches $\bar{u}$, meaning that the derivative of profit will be negative in p_s at that point. ■

Proof of Proposition 2

The proofs of Parts 1, 2, and 4 are essentially identical to the proofs of the equivalent points in Proposition 1.

For Part 3, note that all consumers for whom $u < \tilde{p}_s$ will either watch with advertising or not watch at all when the subscription is available. Consumers in this subset who do not watch in either equilibrium have identical payoffs. Consumers in this subset who watch in the advertising-only equilibrium either watch with more ads when the subscription is available or they switch to not watching, and their welfare must weakly decrease with the introduction of the subscription. ■

Proof of Lemma 5

Take the derivative of profit with regard to the price of advertising in the advertising-only model:

$$
\begin{aligned}
\frac{\partial \pi_{plat}}{\partial p_a} =& (1 - H(p_{ad}) G(t^*) F(\frac{\tilde{p}_s}{i(p_{ad})}) \\
& - h(p_{ad})(p_a - w_a) \left(F(\frac{\tilde{p}_s}{i(p_{ad})}) - f(\frac{\tilde{p}_s}{i(p_{ad})}) \frac{\tilde{p}_s}{i(p_{ad})} \right) \\
& * \left[G(t^*) - w_a \frac{(1 - H(p_{ad})) F(\frac{\tilde{p}_s}{i(p_{ad})})}{g'(t^*)} \frac{c^2}{(w_a(1 - H(p_{ad})) F(\frac{\tilde{p}_s}{i(p_{ad})}))^3} \right]
\end{aligned}
\tag{70}
$$

Setting this derivative equal to 0 gives the following FOC

$$
\begin{aligned}
& \frac{1 - H(p_{ad})}{h(p_{ad})} F(\frac{\tilde{p}_s}{i(p_{ad})}) G(t^*) = \\
\frac{p_a - w_a}{v} & \left(F(\frac{\tilde{p}_s}{i(p_{ad})}) - f(\frac{\tilde{p}_s}{i(p_{ad})}) \frac{\tilde{p}_s}{i(p_{ad})} \right) \left[G(t^*) - w_a \frac{(1 - H(p_{ad})) F(\frac{\tilde{p}_s}{i(p_{ad})})}{g'(t^*)} \frac{c^2}{(w_a(1 - H(p_{ad})) F(\frac{\tilde{p}_s}{i(p_{ad})}))^3} \right]
\end{aligned}
\tag{71}
$$

Using Equation (17) to rearrange the advertising price FOC

$$
\frac{1 - H(p_{ad})}{h(p_{ad})} F(\frac{\tilde{p}_s}{i(p_{ad})}) G(t^*) = \frac{G(t^*)}{v} \left(F(\frac{\tilde{p}_s}{i(p_{ad})}) - f(\frac{\tilde{p}_s}{i(p_{ad})}) \frac{\tilde{p}_s}{i(p_{ad})} \right) [p_a - w_a + w_a] \tag{72}
$$

Canceling like terms

$$
\frac{1 - H(p_{ad})}{h(p_{ad})} F(\frac{\tilde{p}_s}{i(p_{ad})}) = \left(F(\frac{\tilde{p}_s}{i(p_{ad})}) - f(\frac{\tilde{p}_s}{i(p_{ad})}) \frac{\tilde{p}_s}{i(p_{ad})} \right) p_{ad} \tag{73}
$$

Divide through by $F(\frac{\tilde{p}_s}{i(p_{ad})})$

$$
\frac{1 - H(p_{ad})}{h(p_{ad})} = \left(1 - \frac{f(\frac{\tilde{p}_s}{i(p_{ad})})}{F(\frac{\tilde{p}_s}{i(p_{ad})})} \frac{\tilde{p}_s}{i(p_{ad})} \right) p_{ad} \tag{74}
$$

From log concavity of $h(\cdot)$,the left hand side will approach 0 as p_a increases, and Assumption 1 and log-concavity of $f(\cdot)$ imply that the right hand side will be strictly positive and increasing. Profit is increasing if the left hand side is greater than the right, and decreasing if it is less, so these conditions are sufficient for quasi-concavity. There must be at most one solution to the FOC above and it is sufficient for profit maximization. If the profit maximizing price is at a corner, it must be at $\underline{z}$ and not $\bar{z}$ because $p_a = \bar{z}$ would imply 0 mass of advertisers which would in turn lead to 0 profit.

In the subscription model the advertising pricing equation will again be given by taking the derivative of profit with regard to the ad price

$$\frac{\partial \pi_{plat}}{\partial p_a} = i(p_a)F(\frac{\tilde{p}_s}{i(p_{ad})}) - h(\tilde{p}_a)\left[p_a F(\frac{\tilde{p}_s}{i(p_{ad})}) + (\tilde{p}_s - i(p_a)p_a)\frac{\tilde{p}_s}{i(p_a)^2}f(\frac{\tilde{p}_s}{i(p_{ad})})\right] \tag{75}$$

and setting it equal to 0

$$\frac{1-H(\tilde{p}_a)}{h(\tilde{p}_a)}F(\frac{\tilde{p}_s}{i(\tilde{p}_a)}) = \left(\frac{\tilde{p}_s}{i(\tilde{p}_a)}\right)^2 f(\frac{\tilde{p}_s}{i(\tilde{p}_a)}) + p_a\left(F(\frac{\tilde{p}_s}{i(\tilde{p}_a)}) - \frac{\tilde{p}_s}{i(\tilde{p}_a)}f(\frac{\tilde{p}_s}{i(\tilde{p}_a)})\right) \tag{76}$$

Assumption 1 and Assumption 4 together ensure that increasing p_a will decrease the left hand side relative to the right, which again gives quasi-concavity of profits in the advertising price and sufficiency of the first order condition for profit maximization. This is almost identical to the advertising-only FOC, except for the additional term $\frac{\tilde{p}_s}{i(p_{ad})}^2 f(\frac{\tilde{p}_s}{i(p_{ad})})$ which represents that consumers who switch to free viewing as a result of the increased advertising price lowering the advertising level. Therefore, with an interior solution the platform will have a lower advertising price and higher advertising level than when the subscription was not available.■

Proof of Proposition 3

The proofs of parts 1 and 2 are essentially identical to the proof of Proposition 1

For part 3, each advertiser's total profit is

$$G(\hat{t}^*)F(\frac{u}{i(\hat{p}_a)})(z - \hat{p}_a) \tag{77}$$

in the advertising-only equilibrium and

$$G(\tilde{t}^*)F(\frac{\tilde{p}_s}{i(\tilde{p}_a)})(z - \tilde{p}_a) \tag{78}$$

in the equilibrium with the subscription. From Lemma 5, $\tilde{p}_a > \hat{p}_a$ and $G(\tilde{t}^*) > G(\hat{t}^*)$, but $F(\frac{\tilde{p}_s}{i(\tilde{p}_a})) < F(\frac{u}{i(hatp_a})$. The relative size of the effects depends on the $f(\cdot)$ and $g(\cdot)$, so profits can increase or decrease. ■

Figures

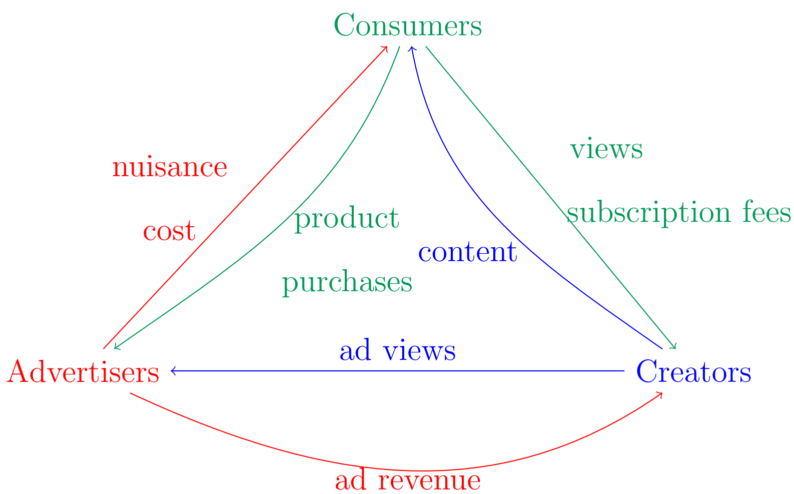


Figure 1: The sides and network externalities in the model

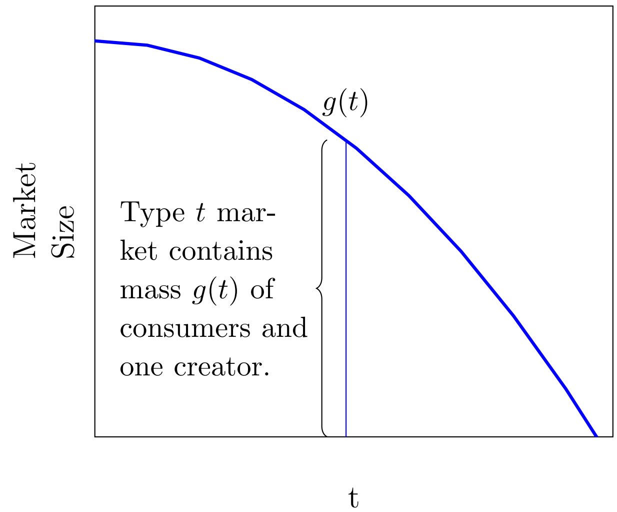
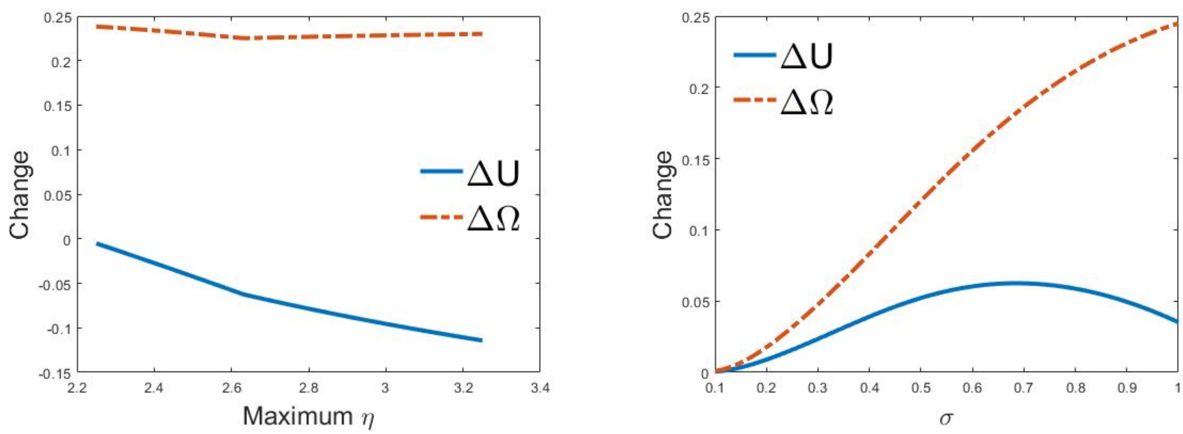


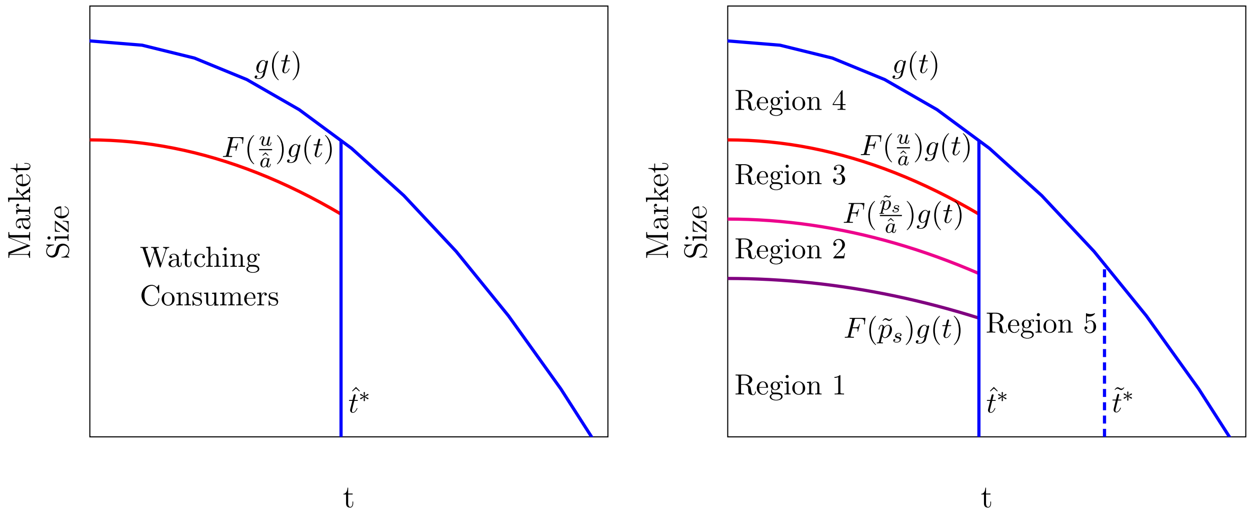
Figure 2: Content market structure



(a) Triangular distribution of nuisance costs

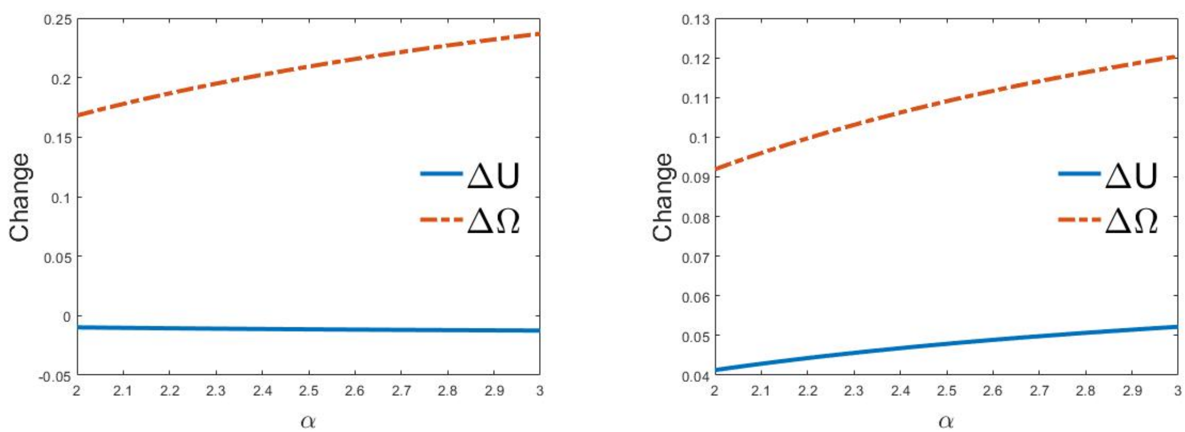
(b) Half-normal distribution

Figure 4: Variation in total welfare and utility by advertising sensitivity. Parameter values: $\underline{\eta} = 0.25, z = 0.2, c = 0.05, \alpha = 3, u = 1$



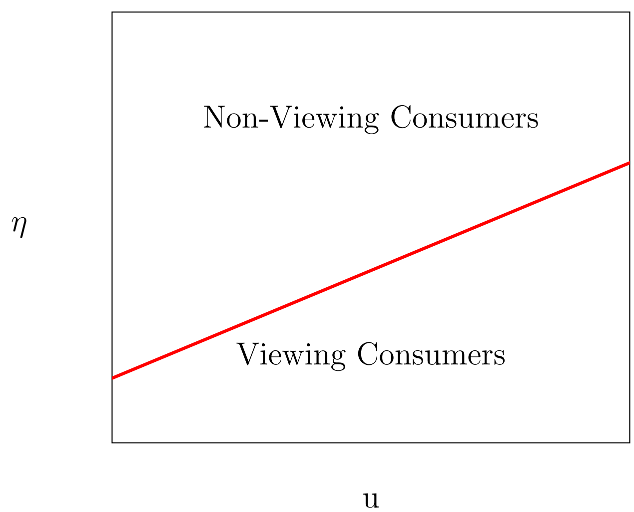
(a) Market coverage in the advertising-only model (b) Market coverage with the addition of the subscription

Figure 3: Market capture

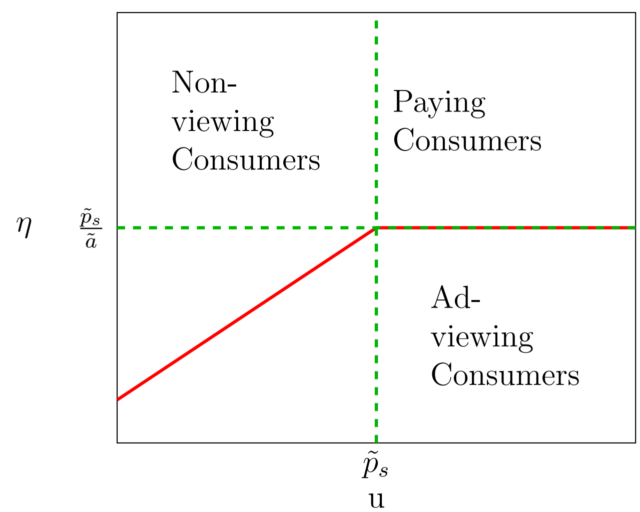


(a) Triangular distribution of nuisance costs (b) Half-normal distribution

Figure 5: Change in total welfare and utility by curvature of $g(\cdot)$.
Parameter values: $\underline{\eta} = 0.25, \mu = 0, z = 0.2, c = 0.05, \bar{\eta} = 2.3, \sigma = 0.5, u = 1$



(a) Advertising-only



(b) With subscription

Figure 6: Consumer decisions as a function of type