How Much Does Schooling Disutility Matter?*

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Abstract

Researchers studying education decisions are often inconsistent in choosing whether and how to include disutility of education. We show that adding a disutility term in an education choice model is equivalent to assuming a relationship between wealth, risk, and education decisions. Disutility of schooling creates a gap between the increase in *utility return* from education and the *financial return*, meaning that agents may avoid schooling even if it would increase their lifetime wealth. Moreover, utility gains from education are decreasing in initial wealth and increasing in riskiness of future consumption. If the degree of risk increases heterogeneously across human capital investment options, then risk aversion and the precautionary savings motive can compound or negate each other depending which option has a greater increase in risk. Our results also explain recent empirical findings, including a relationship between wealth and education, working between periods of schooling, and college major choices.

JEL classification: D11, I2, J24.

Keywords: structural estimation, human capital theory, disutility of schooling, education choices, precautionary saving

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1 Introduction

Many structural models include a utility cost when an agent attends school (e.g. Macdonald, 1981; Card, 1994, 2001; Johnson, 2013; Hai and Heckman, 2017; Cai and Heathcote, 2018; Guo, 2018; Abbott et al., upcoming).[1] However, there is little consensus among researchers about how and when to introduce this disutility term except that it is often crucial for matching observed patterns in education choices. Few papers have examined the theoretical implications and consequent implicit structural limitations imposed by introducing a disutility of education term to agents' decision problems. We aim to fill this gap by creating a simple two-period decision model comparing education decisions with and without the disutility term.[2]

The primary message of this paper is that including disutility of education in a dynamic model is equivalent to assuming a specific relationship between wealth, precautionary saving and education choices. We define the net *financial return* to education as the lifetime budget constraint of the agent if she goes to school minus her budget should she work in the first period. The net *utility return* to education is defined analogously to the financial return, but using the difference between expected lifetime utilities given the two educational choices.[3] Disutility of schooling disrupts an equivalence between positive net financial returns to education and positive net utility returns to education. When schooling affects utility outside of consumption (via education disutility) then it is possible that education could increase income, but not sufficiently for the difference in consumption to overcome the education disutility. Education choices depend on the magnitude of gains in consumption utility from having increased human capital. All else equal, the utility return to education is decreasing in wealth and for an agent with precautionary saving motive it is also increasing in risk. These effects are important as they can place significant restrictions on the behavior of agents in counterfactual simulations, just as using a Gumbel distribution of random utility for demand estimation imposes a specific substitution pattern between products unless the researcher takes steps to impose flexibility (Berry, Levinsohn and Pakes, 1995).

Our most novel theoretical result is that a risk averse agent with a precautionary saving motive (Leland, 1978) is more likely to seek education during periods of high volatility, *if a disutility term is present*. Education serves as a saving/insurance device that transfers current wealth into human capital for future earnings. An agent with a precautionary saving motive will save more of her income when risk increases. Because of this increased saving, the increase in expected consumption utility resulting from any increase in lifetime income is larger when the variance of future consumption is higher. If investing in human capital

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[1] Cunha et al. (2006) demonstrate that the higher the human capital, the easier it is to gain more via education. They also show that human capital investment is time sensitive. Learning in school may become more difficult after passing a certain age. These effects can thought of as "disutility" of schooling. Johnson (2013) models disutility (or psychic cost) of schooling as a function of ability and age of attendance. Hai and Heckman (2017) model disutility based on cognitive and noncognitive ability, type of schools, and full time or part time enrollment status. Abbott et al. (upcoming) further adds family wealth related information to the estimation.

[2] For the purposes of this paper we are agnostic as to whether students actually experience this disutility. Our primary motivation is to highlight the consequences of including or excluding it.

[3] For the sake of brevity we drop "net" from these terms unless it is relevant.

yields a greater financial return than risk free bond savings — the alternative saving vehicle in most standard models — then an increase in consumption variance implies an increase in the utility return to education and the agent is more likely to go to school. Without a disutility term, this increase in the *utility return* of education does not matter because the agent will just choose whichever education option maximizes her lifetime budget constraint.

Risk aversion implies that the utility for an option decreases in the level of risk associated with it. If, for example, the variance of consumption is higher after schooling instead of working in the first period, then schooling will be less attractive than if it had the same risk as working. However, if both schooling and working experience an increase in risk, but to different degrees, then agents will increase their precautionary saving differently for the different options. Two forces affect education decisions in this environment: risk aversion reduces the appeal of the option with higher relative risk, while a rise in precautionary saving increases the importance of a higher financial return. Focusing on the case where schooling has a positive financial return, if working becomes riskier relative to schooling then both forces compound and the utility return to education increases further. If the relative risk of schooling increases, but only moderately, then the precautionary saving motive dominates the effects of risk aversion and attending school still becomes more appealing despite the increase in relative risk. Even if the increase in risk after schooling is sufficiently large so that the risk aversion factor is dominant, its influence will be partially counteracted by the precautionary saving motive.

Similar to Belley and Lochner (2007), concave consumption utility means that the utility returns to education decrease in wealth. The difference between consumption with and without education is constant, but the proportional difference in consumption is smaller with higher base wealth. Consequently, holding human capital constant, a wealthier individual who faces disutility of education will be less inclined to go to school than one currently holding no assets.

The above results demonstrate that adding a disutility term in a structural education model embeds these connections between wealth, risk, and education into any counterfactual simulations. Causal inference requires a clear identification of which results are assumed and which come from the data. Using education disutility without taking these effects into account can lead to mis-classifying implicit assumptions in the model as empirical findings.

Similarly, researchers should maintain consistent assumptions between their estimation or calibration techniques and counterfactual analysis related to the disutility term. Many recent structural papers (e.g. Hai and Heckman, 2017) estimate the disutility term of their models using ability and education related data but not including wealth data. We provide evidence that such estimates are biased when the analysis concerns counterfactual wealth related factors. Because their estimated disutility term does not account for the relationship between wealth and education decisions, any counterfactual analysis will show an incomplete effect of initial wealth.

In addition to their methodological significance, the above connections from risk and wealth to human capital investment through disutility provide a novel alternative explanation for intermittent schooling choices documented by recent empirical literature (e.g. Light, 1995*a*,*b*; Monks, 1997; Dynarski, 1999; Seftor and Turner, 2002; Jepsen and Montgomery, 2012; Johnson, 2013; Arcidiacono et al., 2016; Yang, 2017). Changing wealth/human capital conditions, relative disutility of schooling, and relative risk between work and education can induce agents to enter or exit school. Our results can also be extended to a variety of choices. As an example, we compare college major choices. A larger financial return is required to induce an agent to choose a major with higher risk or larger disutility. This extension adds a new angle to the literature on the Roy Model of college major choice by suggesting sorting based on differential risks and disutilities across choices and agents as well as financial return.

1.1 Related Literature

Our findings on the relationship between initial wealth and education are similar to Belley and Lochner (2007), but we relax a number of their simplifying assumptions (e.g. assuming that the product of the discount rate and one plus the interest rate is 1) which allows us to explore the comparative statics of the model in richer detail. They impose these assumptions because their main focus is on the impact of credit constraints, while we are interested in exploring the relationship between disutility of schooling and schooling decisions. Additionally, Belley and Lochner's model does not include risk, and we show that the variance of future consumption has a significant impact on education decisions *if a disutility term is present.*

There is little consistency across papers in whether and how structural papers examining human capital acquisition parameterize and in estimate their disutility term. For example, Huggett, Ventura and Yaron (2011) and Lee and Seshadri (2019) do not include a disutility of education term though there is human capital acquisition in their models. Blundell et al. (2016) embed a utility cost term and estimate it using parental income related factors. Johnson (2013), Hai and Heckman (2017), Guo (2018), and Abbott et al. (upcoming), however, only rely on ability and related characteristics in the estimation of their disutility terms. Our results show the importance of estimating disutility using information from both wealth and human capital related data moments in order to avoid omitted variable bias and bias in counterfactual analysis on the impact of wealth in human capital investment choices.

Levhari and Weiss (1974) and Bilkic, Gries and Pilichowski (2012) examine the relationship between risk and education choices, but without differentiating the utility cost from the other costs. Our results demonstrate that disutility of schooling connects wealth and precautionary saving motives to education choices, providing some guidance for the construction of structural models embedding the disutility term. Recent empirical studies explain varying educational choices using heterogeneous risk aversion and risk levels for individuals with different demographic backgrounds (Brodaty, Gary-Bobo and Prieto, 2014; Schweri, Hartog and Wolter, 2011; Jung, 2015; Hartog and Diaz-Serrano, 2015; Belzil and Leonardi, 2007; Mazza and Ophem, 2018; Mazza, van Ophem and Hartog, 2013; Heckman and Montalto, 2018; Chen, 2008). We show that, when a disutility term is present, heterogeneity in wealth alone can create a difference in education choices. Studies estimating risk aversion and education choices may be vulnerable to omitted variable bias if they specify the utility function without a disutility term.

Additionally, almost all of this previous work has focused on the implications of risk aversion for education decisions without considering the implications of the third derivative of the utility function. Our discussion of Leland's (1978) precautionary saving motive is new, and allows us to make more specific predictions for the interaction between risk and education choices than theoretical models which consider risk aversion alone.

2 Model

We create an individual decision model rather than a general equilibrium model in order to focus on the implications of schooling disutility for education decisions. The decision maker is an agent with a two period lifetime. She is endowed with a certain amount of human capital (h_1) and initial wealth (s_1). At the start of the model ($t = 1$), she decides whether to work or to attend school in addition to optimally choosing consumption (c_1) and savings (s_2) facing consumption price 1 and interest rate r.[4] We normalize the effective wage to 1 as a change in wage does not qualitatively affect our results. If she works then she receives income according to the efficiency units of labor she supplies (i.e. h_1, as wage is normalized to 1). Their human capital remains constant across periods ($h_1 = h_2$). If she chooses to attend school then she must pay tuition κ, and allocate the rest of consumption and saving/borrowing from endowment s_1 and second period income h_2. Second period human capital after schooling increases according to $h_2 = g(h_1)$, where $g(h_1)$ is a continuous, increasing, and concave function following Cunha and Heckman (2007). To represent uncertainty about the future, the agent faces an exogenous shock ϵ to consumption realized only in the second period.[5] Because of the two period lifespan, the agent will always work in the second period, receiving income h_2 in addition to (potentially negative) savings from period one $(1 + r)s_2$.

2.1 Baseline model

In the baseline model, we assume the agent encounters no disutility from schooling. The agent's problem is then to maximize lifetime utility considering the increasing, concave, and twice continuously differentiable consumption utility function $u(c)$ and discount rate β.

$$\begin{gathered}\max u(c_1) + \beta E_\epsilon u(c_2 + \epsilon) \\ \text{s.t.} \\ (1 + r)c_1 + c_2 \leq (1 + r)B_1 + B_2 \\ c_1, c_2 \geq 0\end{gathered}$$

where

$$B_1 = \begin{cases} s_1 - \kappa & \text{if school} \\ s_1 + h & \text{if work} \end{cases}$$

and

$$B_2 = \begin{cases} g(h_1) & \text{if school} \\ h_1 & \text{if work} \end{cases}$$

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[4] We could theoretically allow for a continuous education vs. work decision, but this would require more assumptions about the return to education based on time spent schooling and would not add much to our results.

[5] We do not include bankruptcy in this model, so this article contains an implicit assumption that the range of epsilon is small enough relative to s_1 and h_1 that bankruptcy is not a concern.

B_1 and B_2 are the net income in the first and second period respectively. Together they determine the agent's lifetime budget constraint. We assume that an agent who is indifferent will choose to attend school rather than working. Because $u(\cdot)$ is increasing, it is trivial to show that the budget constraint will hold with equality. Proposition 1 follows directly from the statement of the maximization problem:

Proposition 1. *In the baseline model, an agent will choose to attend school in the first period if and only if doing so maximizes lifetime income. This holds if*

$$g(h_1) - h_1 \geq (1+r)(\kappa + h_1) \tag{1}$$

Proof. All proofs are provided in the appendix. □

The left side of Equation 1 describes the marginal gain of human capital from schooling, and the right side describes the direct tuition cost and the opportunity cost of lost wages and interest earnings. When Equation (1) is satisfied then the financial return to education is positive. As we will see in Section 2.2, with no disutility term the utility return to education must be positive whenever the financial return is positive.

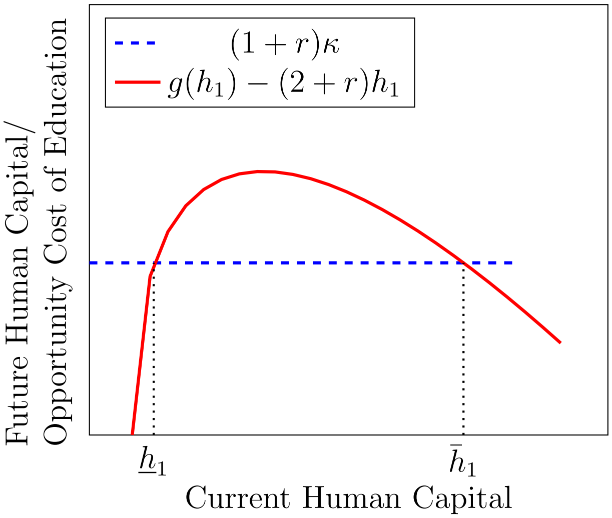

Figure 1: The return to education compared to the opportunity cost.

Rearranging Equation 1 clarifies the comparative statics for the education/work decision.

$$g(h_1) - (2+r)h_1 \geq (1+r)\kappa \tag{2}$$

As $g(h_1)$ is concave, the left hand side is non-monotonic in h_1, increasing for small h_1 before eventually reaching a maximum and decreasing. This relationship is illustrated in Figure 1. For $h_1 < \underline{h}_1$, the agent is not giving up much in the way of wages in order to attend school, but returns to schooling are also small so attending school is not worthwhile. For $h_1 > \bar{h}_1$, the returns are large, but the forgone wages are also large, so the agent prefers to work. The agent will seek education if and only if $h_1 \in [\underline{h}_1, \bar{h}_1]$. As κ increases, the minimum gain in

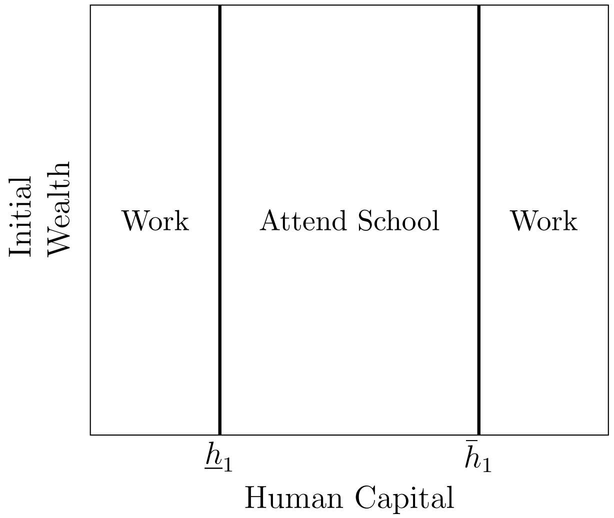

Figure 2: Education decisions as a function of initial wealth and human capital in the baseline model

human capital above which the agent finds it worthwhile to attend school increases, so the range of human capital in which an individual will choose to attend school shrinks.

Wealth does not enter the critical condition in Equation 1.[6] Figure 2 illustrates the education decision as a function of wealth and human capital. Because initial wealth does not impact this decision in the baseline model, the agent will choose to attend school if $h_1 \in [\underline{h}_1, \bar{h}_1]$ regardless of the initial starting wealth.

As we show in the next section, this result is not robust to the introduction of disutility. Independence from wealth depends on the fact that the magnitude of the increase in utility from a positive financial return does not matter. Additionally, if there is a borrowing limit, then wealth independence may not hold. The agent cares about total lifetime wealth only if she can transfer wealth across periods. If she cannot borrow against high future earnings or if doing so would put her at risk of running into a budget constraint from a particularly low ϵ in the second period then she may prefer to work in the first period even though doing so reduces her lifetime wealth.

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[6] If we were to allow the interest rate and/or the tuition to be functions of wealth, then the financial return to education would increase in wealth if and only if $\frac{-r'(w)}{1+r(w)} < \frac{k'(w)}{k(w)+h_1}$, with the return decreasing in wealth if the inequality is reversed. However, this would not impact the intuition behind our subsequent results so we do not directly model this factor.

2.2 Model with disutility of schooling

Following Johnson (2013), we introduce disutility of schooling $d > 0$ to the utility function if the agent chooses school.[7][8] The agent's problem is now

$$\begin{aligned} &\max u(c_1) + \beta E_\epsilon u(c_2 + \epsilon) - \mathcal{I}(\text{Attend School})d \\ &\text{s.t.} \\ &(1+r)c_1 + c_2 \leq (1+r)B_1 + B_2 \\ &c_1, c_2 \geq 0 \end{aligned}$$

where $\mathcal{I}$(Attend School) is an indicator function representing the education decision and B_1 and B_2 are defined as before. Let c_t^e represent period t consumption if the agent chooses to attend school and c_t^w consumption if the agent works in the first period. These consumption choices will be determined by the an Euler equation coming from the first order condition of the maximization problem (see Lemma 1 in the appendix) and the budget constraint given education. The agent will choose to attend school if the utility from doing so is greater than the utility from working. Mathematically:

$$u(c_1^e) + \beta E_\epsilon u(c_2^e + \epsilon) - d \geq u(c_1^w) + \beta E_\epsilon u(c_2^w + \epsilon)$$

Rearranging

$$d \leq [u(c_1^e) - u(c_1^w)] + \beta \left[E_\epsilon u(c_2^e + \epsilon) - E_\epsilon u(c_2^w + \epsilon)\right] \tag{3}$$

The right hand side of Equation (3) represents the change in consumption utility from choosing to attend school. The agent will go to school if and only if this return is greater than the disutility d of attending school. If Equation (1) is not satisfied, then this difference is negative and the individual will not attend school. However, if Equation (1) is satisfied, but close to equality, then the difference can be positive but not sufficient to satisfy Equation (3); the increase in consumption utility in this case is not sufficient to compensate for the distutility of schooling, so the net utility return is negative even though the financial return is positive.

Proposition 2. *If schooling increases lifetime wealth, then the utility returns to education are decreasing in initial wealth and increasing in patience. Mathematically, if Equation* (1) *is satisfied then the right hand side of Equation* (3) *is decreasing in* s *and increasing in* β*.*

This implies that, *holding human capital constant*, the financial returns to education must be greater for a wealthier individual to find schooling worthwhile, but can be smaller for a more patient individual. If $d = 0$ then the comparative statics above would still hold, but in

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[7] Disutility is constant here, whereas it is generally assumed to be a decreasing function of human capital in the empirical literature. Imposing such a relationship would not significantly change our results. See Cunha et al. (2006) for a comprehensive account of the labor literature on lifecycle human capital production. Similarly, wealth can change in-school consumption, but there is no intuitive reason why it should directly impact disutility of schooling. If we were to allow disutility to vary with human capital, then we would expect disutility to be negatively correlated with wealth via a positive correlation between initial human capital and wealth.

[8] Our results carry through in sensible ways if we allow $d < 0$, but this detail does not add much insight and so we make this restriction to simplify the statement of the propositions.

a sense would not matter as Equation (3) is trivially satisfied whenever education increases lifetime wealth. It is only when there is a disutility term that the utility return to education (and hence comparative statics regarding it) have any impact on agents' decisions separate from the implications of the financial return. The range of human capital in which agents will choose to attend school is narrowing as s increases and widens in β. The relationship between the critical range of h and wealth is illustrated in Figure 3. Agents are just indifferent between working and education at $\underline{h}_1$ or $\bar{h}_1$ with 0 starting wealth, but will strictly prefer to work for any $s > 0$.[9]

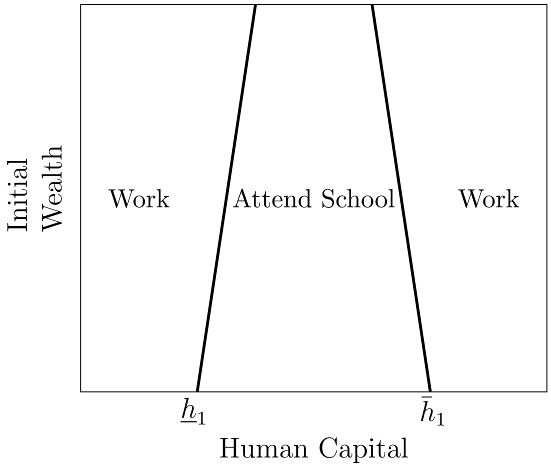

Figure 3: Education decisions as a function of initial wealth and human capital in the model with disutility of education

As mentioned in Footnote 8, we can accommodate a situation where the agent derives positive utility from school (e.g. if there is some "joy of learning"). In this case the agent would attend school even if it created a financial loss, with the size of the loss required to induce the agent to work in the first period increasing in s. We illustrate this scenario in Figure 4.

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[9] Note that the symmetry and linearity in Figure 3 are not guaranteed. The actual cutoff points depend on the utility function and d. The figure is merely a non-parametric illustration.

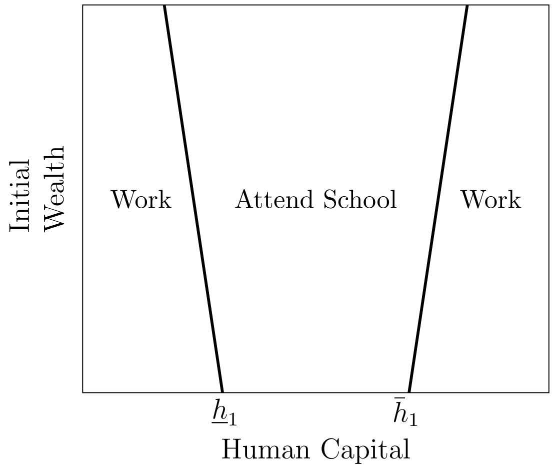

Figure 4: Education decisions as a function of initial wealth and human capital in the model with $d < 0$

Figure 3 seems to be at odds with empirical evidence. There exists a well documented and growing education gap between wealthy and poor households (Pfeffer, 2018; Myong, 2016). However, recall that this is an individual decision model. Depending on the density of individuals in the parameter space, it is entirely possible to have a positive correlation between wealth and education despite the greater financial returns necessary to make schooling desirable. Yang (2017) provides empirical evidence for a positive correlation between initial wealth and human capital. Following this pattern, Figure 5 illustrates a hypothetical situation where human capital and initial wealth are positively associated. At the low end of the wealth distribution, few agents attend school because their human capital is below $\underline{h}$. As wealth increases, a growing proportion of individuals fall between the critical values. If we were to further add a binding borrowing constraint along with highly skewed wealth inequality, then this positive association would be even stronger.[10]

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[10] De Nardi and Fella (2017) document highly skewed wealth inequality. With fewer households on the right tail of initial wealth. Therefore we would expect relatively few agents to fall in the upper right portion of the parameter space and a relatively high proportion in the lower left or center.

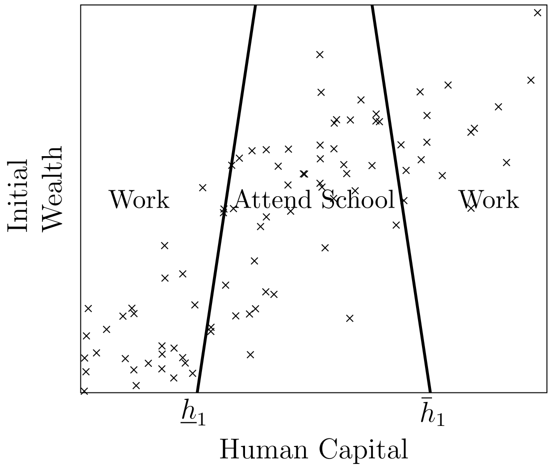

Figure 5: Education decisions with positively correlated initial wealth and human capital

Remark 1. *The choice to make* ϵ *additive rather than multiplicative does not significantly affect our results. Most of the proofs (See Remark 3) carry through with the exact same logic if utility in the second period is given by* $u(\epsilon c_2)$.

Remark 2. *A similar equivalence holds between second period consumption risk and second period income risk (i.e. if* ϵ *were to enter as* $B_2 + \epsilon$ *or* ϵB_2*).*[11] *The reason is that because the agent does not save anything in the second period, any change in the second period budget constraint is exactly equivalent to a change in second period consumption, and thus influences the agent's saving decisions in precisely the same manner. The equivalence between income and consumption risk may not hold in an infinite horizon model or if we were to allow for bankruptcy, but exploring this difference is beyond the scope of this paper.*

Remark 3. *While the broad results (including all of Section 2.2) do not change for multiplicative risk, moving from additive to multiplicative risk means that the risk profiles are no longer equivalent between options. With multiplicative risk the option which maximizes lifetime wealth has a higher variance in outcomes because* c_2 *is higher. We focus on additive risk for the main model because it allows us to cleanly separate the implications of first moment vs. second moment comparative statics, but expand upon this point in Section 3.1*

3 Risk aversion, precautionary saving and education

Given each education choice, consumption/saving decisions will be the solution to an Euler equation derived from the first order conditions of the utility maximization problem. Consider the agent's Euler equation for saving if she chooses to attend school:

$$u'(s_1 - s_2 - \kappa) = \beta(1+r)E_\epsilon u'((1+r)s_2 + g(h_1) + \epsilon)$$

Since $u(\cdot)$ is concave, saving must decrease as $g(h_1)$ increases. This is because human capital acts as a substitute for monetary saving via higher future income. In an empirical setting,

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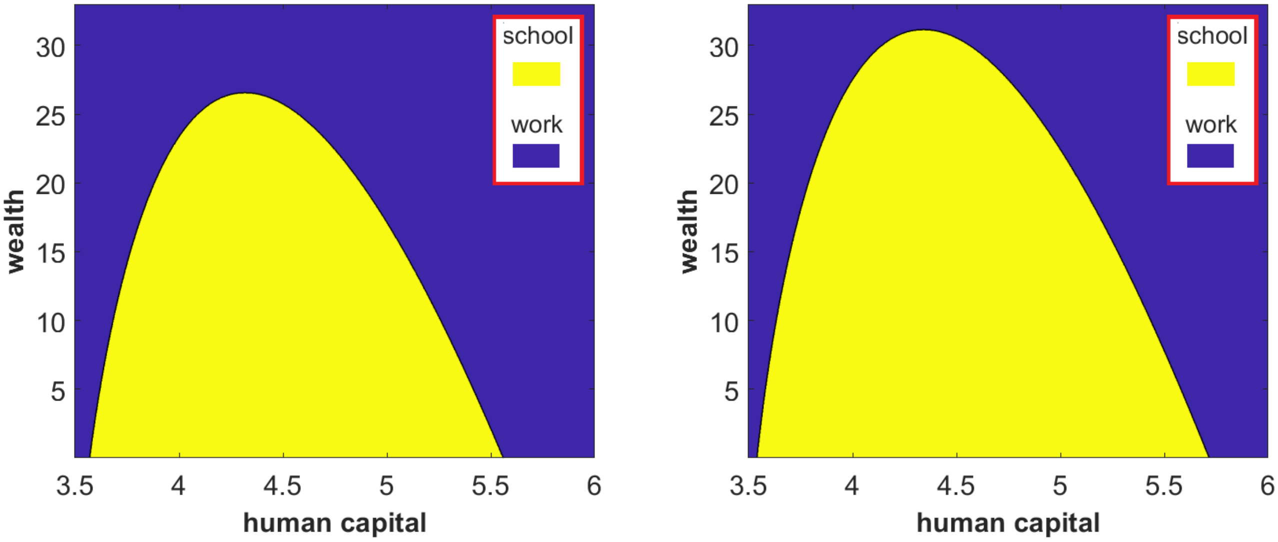
[11] Note that because this is a two period model, second period income risk is equivalent to second period human capital risk.



Low Risk High Risk

Figure 6: Schooling decisions under risk. ϵ has lower variance in the graph on the left. Parameter values: $\gamma = 6$ $d = 0.01$ $\kappa = 0.5$ $r = 0.04$ $\beta = 0.98$ $\theta = 3$ $\alpha = 0.9$ $\epsilon_{\text{high risk}} = [-2.5, 2.5]$ $\epsilon_{\text{low risk}} = [-2.2, 2.2]$ utility function: $c^{(1-\gamma)}/(1-\gamma)$ human capital production: $g(h_1) = (\theta * h_1)^{\alpha}$

this could mean that what appears to be borrowing behavior (spending money in order to attend school) is in fact driven by a desire to save. Additionally, if marginal utility is convex (meaning that the agent engages in precautionary saving), and there is disutility of schooling, then it is possible for an increase in risk (via increased variance of ϵ) to increase the likelihood an agent goes to school. The increased risk will increase the agent's desire to save, which can be done most effectively via education if Equation (1) holds. However, similar to Proposition 2, the change in utility return only matters to the education decision if there is a wedge between the parameter sets where the financial return to education is positive and the sets where the utility return is positive (i.e. when $d \neq 0$).

Proposition 3. *If schooling increases wealth and there is a precautionary saving motive, then the utility returns to education are increasing in the riskiness of the environment. Mathematically, if $u'''(\cdot) > 0$, and Equation* (1) *is satisfied, then applying a mean preserving spread to ϵ increases the right hand side of Equation* (3)*.*

Figure 6 shows the results of a numerical example demonstrating this effect in an environment with a CRRA utility function and risk aversion parameter γ.[12] The brightly colored region represents parameter ranges where an agent would choose to attend school, and the darker region parameters where she would work in both periods. We see the narrowing pattern in wealth from Proposition 2 in both images, but the schooling region is larger in the image on the right where the variance of ϵ is higher. If the agent is working in a field where

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[12] Because we do not model bankruptcy, the parameter ranges in the numerical example have been chosen so that there is no risk of negative 2nd period consumption.

a structural change implies both an increased risk of being fired and the potential for high future earnings (as happened for IT workers in the late 90s) then our results imply that a model simulating this increase in volatility will predict that she responds by investing in her human capital to insure against the downside risk.

3.1 Heterogeneous risk across choices

One of the simplifying assumptions that we have been making throughout this paper is that the risk profile does not depend on the education choice. There are many reasons why risk might vary depending on the human capital investment decision. Krueger and Perri (2006) and Heathcote, Storesletten and Violante (2010) document varying volatility of earnings across different skill types over time. Alternatively Huggett, Ventura and Yaron (2011) and a series of lifecycle structural papers following it introduce multiplicative second period risks, which mechanically means that variance will be higher for whichever human capital investment option has a larger lifetime budget constraint.

Given that the agent has risk averse preferences, it follows immediately that total utility for an option is decreasing in the level of risk associated with that option (as measured by the mean preserving spread definition of Rothschild and Stiglitz (1970)). Therefore if the risk associated with the schooling option is lower than the risk after working then this will make education more appealing relative to working for any parameter set.

Remark 4. *Proposition 3 does not apply when the riskiness of only one option has increased. In that scenario the agent is only saving more of her income if she chooses the option with the increased risk. The utility from consumption in the other option is unchanged. The intuition behind the proposition (that the financial return to education becomes more important as the riskiness of the environment increases) only applies if the agent is saving more of her income no matter her educational choice.*

While the implications of increasing the riskiness of one option are straightforward, the precautionary saving results from Proposition 3 can still apply when the riskiness of both options increases, but by differing amounts (as can happen when risk is multiplicative). Concave utility implies that the option which has the relative increase in risk loses appeal, while the increase in precautionary saving across both options implies that the financial returns to education (if they are positive) become more important. If Equation (1) is satisfied and working becomes riskier compared to schooling, then the precautionary saving motive and risk aversion will compound and the effect from proposition 3 will be stronger than if the increase in risk is symmetric. More interestingly, if the riskiness of schooling increases more than the risk of working, but the difference is not too large, then the precautionary saving result will dominate and the appeal of school will still increase. To formalize this define $\epsilon^i \overset{d}{=} \epsilon + \sigma^i * z$ where z is a random variable with mean 0, $i \in \{e, w\}$ and $\sigma^i > 0 \; \forall \; i$. ϵ^i is the mean preserving spread applying to choice i, with σ^i representing the magnitude of the increase in variance.

Proposition 4. *If the risk of both options increases, and the risk of schooling does not increase by too much more than working, then the precautionary savings result still applies: If $u'''(\cdot) > 0$ and Equation* (1) *is satisfied, then there exists a cutoff $\delta^* > 0$ such that applying* *the mean preserving spreads* ϵ^w *and* ϵ^e *to working and schooling respectively increases the right hand side of Equation* (3) *if and only if* $\sigma^e - \sigma^w < \delta^*$

Proposition 4 implies that the precautionary saving results apply quite broadly. Even if the increase in risk given education is sufficiently large that the relative appeal decreases, this decrease will still be mitigated by the precautionary saving motive.

4 Generalized Human Capital Investment Decisions: An application to major choice

Thus far we have described the environment of this model in the context of a binary choice between schooling and work. However, the results generalize to many scenarios concerning human capital acquisition, such as choosing between career paths or deciding whether to migrate or remain in one's home country. Here, we extend the application to choices between major A or B. The two majors may or may not have different tuition costs, but major B simultaneously provides a higher future wage profile and more disutility than A. Using the superscripts A and B to refer to the parameters/functions associated with majors A and B respectively, this is stated formally as:

$$g^A(h) < g^B(h)\ \forall h$$
$$d^A < d^B$$

By the same logic as in Section 2.2, Major B is preferred if and only if

$$d^B - d^A \leq \left[u(c_1^B) - u(c_1^A)\right] + \beta\left[E_\epsilon u(c_2^B + \epsilon) - E_\epsilon u(c_2^A + \epsilon)\right] \tag{4}$$

By Lemma 1 (see the appendix), consumption is greater if the agent chooses major B as long as κ^A is not drastically lower than κ^B. Both sides of the inequality are positive, so applying the exact same logic as used to prove Proposition 2, the right hand side of Equation (4) is decreasing in initial wealth, meaning that a wealthier agent is more likely to choose major A.

Remark 5. *As demonstrated by the above derivation, Propositions 1 to 3 can be modified in obvious ways to fit this major choice model with major A taking the place of working. Working can be thought of as a special case of major choice, where the human capital development function is the identity function, disutility is 0 and* $\kappa^w = -h$.

Generalizing to n choices is also straightforward. Simply apply the results of the previous sections for each pairwise comparison. For any two choices, an increase in the difference of utilities or the degree of risk between the two options implies that a larger financial return is required to induce the agent to choose the option with more disutility/higher risk.

Saks and Shore (2005) found empirical evidence of a positive relationship between wealth and career risk associated with college major a student chooses. Extending Section 3.1 to college majors provides theoretical support for this finding. Majors with a higher career risk path may generate lower expected consumption utility, but from Proposition 2 consumption utility is less of a factor in the decision making of wealthier individuals, meaning that they are more likely to focus on career preferences (i.e. the utility/disutility of each major) when making education decisions. Cubas and Silos (2017) document positive a correlation between average earnings and risk across industries, and interpret this sorting using ability and risk levels in a Roy Model framework. Our theoretical results suggest another channel: sorting via disutility.

5 Discussion

5.1 Implications for Counterfactual Analysis

Any study which includes a disutility term should keep the implicit assumptions that follow from doing so in mind when interpreting their results. If, for example, a researcher using this setting were to find an increase in school attendance after simulating the effects of policy which increases uncertainty about the future, then that result does not necessarily serve as empirical evidence for a positive relationship between risk and schooling decisions. As we show, it is likely a result of the structure of the model rather than an effect that is coming from the data.

Kemptner and Tolan (2018) conduct a numerical exercise to show that failure to account for time-inconsistent discounting in a structural model may lead to erroneous policy simulations. However, their results rely on simultaneous estimation of both discounting and disutility parameters, without differentiating the impact from the latter. From Proposition 2, disutility and the discounting term do not affect the education decision independently. Therefore, while they effectively demonstrate that ignoring present bias can interfere with policy simulations, their results may be subject to an inconsistent structural assumption similar to the one they were trying to highlight.

In the seminal work by Hai and Heckman (2017), disutility is a key term generating education behavior in their model. They estimate the utility cost of schooling only using data on schooling choices, ability measures, and parental education. In their counterfactual analysis, they find that wealth related factors (including parental net worth, tuition subsidies, and borrowing limits) have limited impact on agents' education choices, consumption, and wage inequality in the lifecycle. Our results provide evidence that estimation of education disutility without introducing wealth information suffers from an omitted variable bias. Counterfactual exercises studying changes in wealth related factors would provide an incomplete and biased conclusion when using these biased estimates.

Alternatively, studies investigating the relationship between wealth and income should consider the impact of not including a disutility term for skill acquisition when it might be present (e.g. Lee and Seshadri, 2019; Krebs, Kuhn and Wright, 2015; Huggett, Ventura and Yaron, 2011). All of our main results rely on the disutility term driving a wedge between financial and utility returns. Dropping it assumes away this channel, which could (for example) lead to an underestimation of the impact of family wealth on education decisions.

5.2 Empirical Implications

While the primary motivation for this paper is to explore the implicit assumptions coming from adding education disutility in structural analysis, these results can serve as explanations for some real world behavior.

Most studies explain delayed college entry using financial costs by arguing that students are either at a borrowing limit or that they expect to be at one if they enroll in college (e.g. Johnson, 2013). Previous investigations into college dropout/stopout/re-entry explain these phenomena via time varying tuition related cost, uncertainty in college premium, and agents' ex ante uncertainty about their ability to complete college (Donovan and Herrington, 2019; Hendricks and Leukhina, 2018; Yang, 2017; Lee, Lee and Shin, 2014). In our model, the utility wedge created by disutility of schooling can vary across time as parameters of the decision problem change. For example, a student who has previously exited college might wish to re-enroll if her expectations about future risk increase or if she experiences a sudden drop in wealth. These factors also predict when a change in attitude (i.e. a change in disutility) will affect re-enrollment/dropout decisions.[13] For example, a wealthier student is more likely to drop out than a poorer one because the range of human capital where she finds schooling still worthwhile is narrower.

There is a significant body of evidence showing that students consider expected returns when choosing their major (Montmarquette, Cannings and Mahseredjian, 2002). Fricke, Grogger and Steinmayr (2018) provide empirical evidence that varying levels of disutility between different fields also leading to major choices. Our results provide a theoretical framework for how this operates.

In the literature of skill investment, the Roy Model predicts that for any given skill distribution and relative price of the skills in the labor market, individuals self-select into majors where they have a comparative advantage based on their expected wages (e.g. Altonji, Blom and Meghir, 2012; Kirkeboen, Leuven and Mogstad, 2016). Our results show that different disutilities for each major choice create a wedge between the financial and utility returns of each major. Different risks associated with each career path drive a second moment precautionary savings motive sorting that is worth further empirical investigation.

6 Conclusion

We present a two-period model where a decision maker decides between schooling and working in the first period in addition to optimizing her consumption/saving choices across periods. If she does not experience disutility from education then she will simply choose the education option which maximizes her lifetime income. This choice does not depend on initial assets, so wealth does not play a role in this decision. On the other hand, if education disutility is a factor then the *utility return* to education is partially disconnected from the *financial return*. This utility return may be negative even if the financial return is positive. Furthermore, the utility return is decreasing in wealth and when there is a precautionary saving motive it is increasing in the variance of future consumption, meaning that introducing

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[13] Recent structural papers measure changing disutility as a function of ability, institution type, age, family income, and some random shocks (e.g. Johnson, 2013; Blundell et al., 2016; Hai and Heckman, 2017).

a disutility term makes education choices much more sensitive to the parameters of the model. When the increase in risk is heterogeneous across options, then risk aversion and the precautionary savings motive can compound or counteract each other, depending on which option has a higher increase in relative risk.

These results imply that adding a disutility term in structural estimation is equivalent to assuming a specific relationship between wealth, risk, and education decisions in counterfactual simulations. Therefore researchers investigating educational choices should carefully consider the implications of including or excluding this distutilty term when interpreting their results.

We can apply our model to explain a number of behaviors seen in recent empirical papers, including a relationship between wealth and education decisions as well as spells of working in between periods of school enrollment. We also add to the traditional Roy Model by suggesting that students may sort based on differential risk and disutilities across choices as well as financial return.

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A Proofs

The following lemma is somewhat obvious, but we find it helpful for later proofs to give the result a concise label:

Lemma 1. *Both c_1 and c_2 are increasing in lifetime wealth. Therefore total utility is increasing in wealth.*

Proof. The agent's period 1 saving/consumption decision is determined by the following Euler equation

$$u'(c_1) = (1+r)\beta E_\epsilon u'(c_2+\epsilon) \tag{5}$$

If c_1 increases (decreases) and c_2 does not, then this equality will no longer hold and the agent would prefer to shift consumption into the future (past). Therefore c_1 cannot increase as lifetime wealth increases without c_2 also increasing. The same logic can be applied to c_2 to complete the first part of the claim.

If both c_1 and c_2 increase, then total lifetime utility must increase as $u(\cdot)$ is an increasing function. Therefore from the first part of the claim total lifetime utility must increase in wealth. □

Proof of Proposition 1

Schooling enters the agent's problem in the baseline model only through its effect on the lifetime budget constraint. From Lemma 1 the agent will make the education choice which maximizes $(1+r)B_1 + B_2$. Schooling maximizes this lifetime budget constraint if and only if

$$(1+r)(s_1-\kappa) + g(h_1) \geq (1+r)(s_1+h_1) + h_1$$

Equation (1) follows immediately from simplifying this equation.□

Proof of Proposition 2

Because $u(\cdot)$ is increasing, the differences on the right hand side of Equation (3) will be positive if and only if the inequality in Equation (1) holds, but the amount by which they are positive will depend on s. Taking the derivative of the right hand side of Equation (3) with regard to s

$$\begin{aligned}&\frac{\partial}{\partial s}\left([u(c_1^e) - u(c_1^w)] + \beta\left[E_\epsilon u(c_2^e + \epsilon) - E_\epsilon u(c_2^w + \epsilon)\right]\right)\\ =&\left[u'(c_1^e)\frac{\partial c_1^e}{\partial s} - u'(c_1^w)\frac{\partial c_1^w}{\partial s}\right] + \beta\left[E_\epsilon u'(c_2^e + \epsilon)\frac{\partial c_2^e}{\partial s} - E_\epsilon u'(c_2^w + \epsilon)\frac{\partial c_2^w}{\partial s}\right]\end{aligned}$$

From Equation (5), this can be simplified to

$$=u'(c_1^e)\left(\frac{\partial c_1^e}{\partial s} + \frac{1}{1+r}\frac{\partial c_2^e}{\partial s}\right) - u'(c_1^w)\left(\frac{\partial c_1^w}{\partial s} + \frac{1}{1+r}\frac{\partial c_2^w}{\partial s}\right) \tag{6}$$

But since the budget constraints must always hold with equality, we can take the derivative of both sides of the budget constraint in the definition of the agent's problem with regard to s to show that $(1+r)\frac{\partial c_1^e}{\partial s} + \frac{\partial c_2^e}{\partial s} = (1+r)\frac{\partial c_1^w}{\partial s} + \frac{\partial c_2^w}{\partial s} = 1 + r$. Dividing through by $1 + r$, this then shows that Equation (6) further simplifies to

$$u'(c_1^e) - u'(c_1^w) < 0$$

Where the inequality comes from Lemma 1, the assumption that Equation (1) is satisfied, and concavity of $u(\cdot)$. To see the relationship with β, note that if β decreases, then the right hand side of Equation (5), will also increase, which by the steps above implies that the utility returns to an increase in wealth also increase.□

Proof of Proposition 3

Define

$$B^e = (1+r)(s_1 - \kappa) + g(h_1) \tag{7}$$

$$B^w = (1+r)(s_1 + h_1) + h_1 \tag{8}$$

These are the budget sets for an agent who chooses education and one who chooses to work in the first period. Define the value function

$$\begin{gathered}V(B) = max_{c_1,c_2} u(c_1) + \beta E_\epsilon u(c_2 + \epsilon)\\ \text{s.t.}\\ (1+r)c_1 + c_2 \leq B\\ c_1, c_2 \geq 0\end{gathered} \tag{9}$$

Then utility in the environment where the agent works is $V(B^w)$, and utility upon receiving education is $V(B^e) - d$. Following theorem 2 in Milgrom and Segal (2002), Equation (3) can be rewritten as

$$d \leq V(B^e) - V(B^w) = \int_{B_w}^{B^e} V'(x)dx \tag{10}$$

Let $c_1(B)$ be the period 1 consumption assigned by the value function with budget B. An immediate corollary of the proof of Proposition 2 is that $V'(B) = u'(c_1(B))$, so

$$\int_{B_w}^{B^e} V'(x)dx = \int_{B_w}^{B^e} u'(c_1(x))dx \tag{11}$$

Now let ϵ^* be distributed according a mean preserving spread of the distribution of ϵ and assume that $u'''(\cdot) > 0$. $c_1(B)$ and the analogous $c_2(B)$ are determined by the new Euler equation

$$u'(c_1(B)) = \beta E_{\epsilon^*} u'(c_2(B) + \epsilon^*)$$

Because $u'''(\cdot) > 0$, Leland's (1978) well known result on precautionary saving gives that $c_1(B)$ will be lower under ϵ^* than ϵ for any B. From concavity of $u(\cdot)$ this implies that $\int_{B_w}^{B^e} u'(c_1(x))dx$ is greater under ϵ^* than ϵ. $\square$

Proof of Proposition 4

Analogously to Proposition 3, define

$$\begin{aligned} V(B,\sigma) = max_{c_1,c_2} u(c_1) + \beta E u(c_2 + \epsilon + \sigma * z) \\ \text{s.t.} \\ (1+r)c_1 + c_2 \leq B \\ c_1, c_2 \geq 0 \\ \epsilon \sim F(\epsilon) \\ z \sim G(z) \\ E(z) = 0 \end{aligned} \tag{12}$$

The value function is now a function of the budget constraint and the distribution of risk. Similarly to Proposition 3, Equation (3) can be rewritten as

$$d \leq V(B^e, 0) - V(B^w, 0) \tag{13}$$

When we impose the mean preserving spread the right hand side becomes

$$V(B^e, \sigma^e) - V(B^w, \sigma^w) \tag{14}$$

Further, from Proposition 3

$$V(B^e, \sigma^w) - V(B^w, \sigma^w) > V(B^e, 0) - V(B^w, 0) \tag{15}$$

Using Jensen's Inequality and the continuous differentiability of $u(\cdot)$, it follows that $V(B,\sigma)$ is decreasing continuously in σ, so let $\delta = \sigma^e - \sigma^w$. If $\delta < 0$ then $\sigma^e < \sigma^w$

$$
\begin{aligned}
V(B^e, \sigma^e) - V(B^w, \sigma^w) &= V(B^e, \sigma^w + \delta) - V(B^w, \sigma^w) \\
&> V(B^e, \sigma^w) - V(B^w, \sigma^w) \\
&> V(B^e, 0) - V(B^w, 0)
\end{aligned} \tag{16}
$$

Alternately, if $\delta > 0$, then by continuity of the value function, we can find δ sufficiently small so that

$$
V(B^e, \sigma^w) - V(B^e, \sigma^e) < V(B^e, \sigma^w) - V(B^e, 0) + V(B^w, 0) - V(B^w, \sigma^w) \tag{17}
$$

Where the right hand side of Equation (17) is positive from Equation (15). This gives us that, for δ sufficiently small

$$
\begin{aligned}
V(B^e, \sigma^e) - V(B^w, \sigma^w) &= V(B^e, \sigma^e) - V(B^w, \sigma^w) + V(B^e, \sigma^w) - V(B^e, \sigma^w) \\
&= V(B^e, \sigma^w) - V(B^w, \sigma^w) - (V(B^e, \sigma^w) - V(B^e, \sigma^e)) \\
&> V(B^e, \sigma^w) - V(B^w, \sigma^w) - (V(B^e, \sigma^w) - V(B^e, 0) + V(B^w, 0) - V(B^w, \sigma^w)) \\
&= V(B^e, 0) - V(B^w, 0)
\end{aligned} \tag{18}
$$

The result follows. □