Membership versus Discovery Platforms*

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Abstract

We model a market where content creators compete for a discovery platform's recommendation via a Tullock contest where effort is measured by the creation of public content. They can also create exclusive content for a membership platform. Both a pure discovery platform and an integrated monopolist always set a high public content commission because discovery is an essential input for creators. Time investment in both content types is determined by the exclusive content commission. The monopolist internalizes this interdependence, leading to greater social welfare. When the discovery platform can change recommendation weights for different creator types then both profits and social welfare are convex in these weights under both market structures. The monopolist is more likely to assign the socially optimal weight as its profits depend on the value of content while a pure discovery platform maximizes the number of views.

JEL classification: D26, D4, L2, L5

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1 Introduction

Content discovery platforms like YouTube, Twitch, Instagram, and more recently TikTok have traditionally made the majority of their money by selling the attention attracted by content creators hosted on their platforms to advertisers. However as online content creation has matured as an industry, membership based crowd-funding platforms like Patreon and Subbable have sprung up allowing creators to solicit direct transfers from their audience in exchange for exclusive content, and some discovery platforms such as Twitch have incorporated similar arrangements

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directly. These subscription models allow audiences to subsidize the creation of content more efficiently than does advertising.[1]

Discovery platforms play a bottleneck role in matching consumers to content creators, while membership platforms play a crucial role in enabling content creators to generate revenue directly from the content they create. As such, any strategic decisions by these platforms significantly affect the content creation behavior and the level of economic surplus generated in the content creation market. In this paper, we focus on the following strategic decisions by platforms: (i) the level of commissions and (ii) the types of content they recommend to audiences. We explore how the market structure and the business models of the platforms affect these strategic decisions. Specifically, how is a pure discovery platform coexisting with a pure membership platform (as was the case with YouTube and Subbable/Patreon for many years, and more recently with Twitter and Substack until Twitter introduced an option to allow for paid tweets) different from a platform like Twitch which incorporates both business models?[2]

To answer this question, we develop a theoretical model where content creators divide their time between making advertising funded public content for a discovery platform and subscription funded exclusive content for a membership platform. The membership platform may or may not be integrated with the discovery platform. The advertising revenue in this model is exogenous, whereas creators set the access price for any exclusive content they might create. Consumers find creators via public content recommended to them by the discovery platform, and once they know of a creator they can choose to purchase access to any exclusive content that the creator might produce. Platform(s) levy ad-valorem commissions on each unit of advertising revenue and each unit of exclusive content revenue.

Content creators can be either "broad" (denoted b creators) where most consumers will like their content but place relatively low value on that content, or they can have "limited" appeal (l creators) where they have a relatively low probability of being liked by any given consumer, but provide a high value if consumers do like them.[3] We model the recommendation process as a weighted Tullock contest for each consumer, where the platform can assign different weights on different types of creators to manipulate the relative probability of each type of creator being recommended.[4]

The main focus of this paper is comparing a hybrid monopolist platform (with both discovery and membership components) to an asymmetric duopoly (with a pure membership platform and a pure discovery platform). Our first result is that a hybrid monopolist will set lower commissions than would asymmetric duopolists. The reason is that discovery acts as an essential input for the content creators, so creator behavior is always inelastic in the face of a change in

1. This efficiency argument comes from the fact that advertising revenue comes from the value of advertisers for ad views, which is usually much lower than the value consumers have for content.

2. Patreon touts their lack of a discovery component as a strength, saying that it allows creators to design their content without worrying about appealing to the recommendation algorithm. https://blog.patreon.com/why-isnt-patreon-discovery-platform

3. This is a binary simplification of Johnson and Myatt's (2006) demand rotation concept. Most of our results hold if we allow for a continuous distribution of consumer values.

4. In practice, creators often complain of having to "chase the algorithm", which suggests that they compete with each other indirectly to get recommended to each consumer, much like a contest. A contest function provides a tractable reduced-form formulation to capture this type of competition. Following the contest literature, we adopt the Tullock contest function due to its desirable analytical properties and strong theoretical foundation (Jia, Skaperdas, and Vaidya 2013).

the commission on public content. Consequently, the public content fee is always quite high, meaning that the revenue from exclusive (membership) content is the main determinant of how much of both types of content get produced. This leads to a negative spillover effect whereby increasing the exclusive content fee reduces the amount of public content created as well as exclusive content. An integrated monopolist will fully internalize this spillover effect because it obtains revenues from both types of content. In contrast, a pure membership platform obtains revenues from exclusive content only and so when it raises its fee it does not take into account the corresponding revenue loss suffered by the pure discovery platform. As a result, the pure membership platform sets a higher commission than would an integrated monopolist, which leads to less content being created and a lower total surplus.

Our second result is that a duopolist discovery platform, relative to an integrated monopolist, is more likely to focus on recommending broad creators when it comes to the design of recommendation algorithms. The duopolist pure discovery platform favors the b creators because they will have the maximum number of views per recommendation and the revenue from public content is purely a function of how many consumers view content and not how much they value it. In contrast, the hybrid monopolist takes into account the revenue from the more valuable exclusive content created by l creators, and as such it is less likely to focus on recommending broad creators. We then compare these designs with the welfare-maximizing design, and show that the hybrid monopolist's choice of design is more likely to be aligned with the welfare-maximization objective under some relatively mild conditions

This paper echoes the recent interest in understanding how different business models of digital platforms can lead to different welfare implications.[5] We highlight that content creation markets feature an important complementarity relationship between the creation of public content and exclusive content because each creator often spends time on both types of content. The hybrid monopolist, given that it obtains revenues from both types of contents, can internalize this complementarity (and the total surplus generated) more than the asymmetric duopolists. Consequently, the monopolist tends to make more welfare-efficient strategic choices (e.g. pricing and algorithm designs) than would duopolists.

2 Related literature

Media firms are one of the earliest examples of a platform business model, although the majority of previous papers have focused on these platforms as intermediaries between consumers and advertisers (Anderson and Coate 2005; Choi 2006; Crampes, Haritchabalet, and Jullien 2009; Anderson and Peitz 2020). There are a number of recent examples which consider independent content creators and how the competitive environment affects their behavior (Kerkhof 2020; Pei and Mayzlin 2019; Fainmesser and Galeotti 2020), but for the most part these papers do not consider platform design, and even when they have they do not cover our specific question of providing content at a free or low price to aid discoverability vs exclusive content with a more effective transfer of value. This tension between ad-supported and paid content is closely related to the literature comparing paid television content vs. "free-to-air" business models (Peitz and

5. See, e.g., Caffarra et al. (2020), Etro (2021), and Teh (2021).

Valletti 2008), but again these papers have generally not considered independent content creators or discoverability.

Two of the motivating examples for membership platforms are Patreon and Subbable, both of which are crowdfunding platforms. Our paper thus has a loose connection to the crowdfunding literature (Deb, Oery, and Williams 2019; Ellman and Hurkens 2019). Notably however, this literature tends to focus on the mechanism design aspects of one-shot project-based crowdfunding projects, whereas the business model of membership platforms is based around support for content creators who produce content on a continuing basis so long as doing so is more appealing than their outside option.

3 Monopoly platform benchmark

There is a platform, a continuum of consumers of measure 1, and $N \geq 2$ content creators. A consumer i's match value with a content creator j is v_{ij}, which is drawn i.i.d across i and j. With probability λ the consumers likes the content creator and realizes a positive match value $v_{ij} = V > 0$, and with probability $1 - \lambda$ the consumer dislikes the content and realizes a negative value $v_{ij} < 0$. Consumers' outside option (e.g. alternative sources of content) has value normalized to zero. Therefore, consumer i prefers creator j's content over the outside option if and only if $v_{ij} = V$.

Content creators. There are two types of content creators. A number N_b of creators have type $\theta = b$ (broad-market creators) while the remaining $N_l = N - N_b$ creators have type $\theta = l$ (limited-appeal creators). Let λ_θ be the probability a consumer likes a type-θ creator's content and $V_\theta > 0$ be the utility per view if the consumer likes the creator's content. Similar to the demand rotations of Johnson and Myatt (2006), we assume that consumers have a high probability to like a broad-market creator's content, but would realize a higher match value with a limited-appeal creator conditioned on liking the creator's content. Formally, this means $\lambda_b > \lambda_l$ and $V_b < V_l$.

Content discovery. Consumers need to discover content creators before they can start viewing or consuming content, and they do so via the platform's recommendation. We model this matching process as a Tullock contest. Specifically, every time a consumer submits a search query, the platform's recommendation algorithm assigns weight to content based on the probability that a consumer will view it. The more public content a creator makes the more "draws" it has in the algorithm. Thus the weight of a creator j in the platform's Tullock contest is $t_j^\eta W(\lambda_j)$, where t_j^η is the amount of public content created by j and $W(.)$ is an arbitrary weighting function based on the probability consumers will like the content. Then, the total probability that a creator j is recommended to any given consumer is

$$D_j(t_j^\eta) = \frac{t_j^\eta W(\lambda_j)}{t_j^\eta W(\lambda_j) + \sum_{k \neq j} t_k W(\lambda)} \tag{1}$$

After a creator j is recommended to a consumer, the consumer is aware of all of a creator's content. The consumer realizes her match value with creator j, and decides whether to watch the creator's content or exercise their outside option. Without loss of generality, we assume that each piece of content by j corresponds to one "view" by consumer i. Therefore, if a creator has amount t_j of content, the expected mass of views by consumer i is $\lambda_j t_j^\eta$.[6]

Each view of public content generates gross advertising revenue η. Following Gabszewicz, Laussel, and Sonnac (2004) and Casadesus-Masanell and Zhu (2010), we assume that η is exogenously determined by a competitive advertising sector. For each unit of advertising revenue, the platform takes a proportional commission r^η, leaving creator's share $1-r^\eta$.

Exclusive content. Some platforms allow content creators to charge viewers for access to exclusive content, in addition for allowing for advertisement.[7] To capture this, we assume that each creator chooses the amount of exclusive content t_j^p they want to produce (in addition to producing public content t_j^η). Each creator also chooses the per-piece access price p_j for access to exclusive content. Recall that each consumer is aware of all of a creator j's content once j has been recommended, which includes any exclusive content j may have produced. The consumer can then choose whether to pay to view the exclusive content, which she does if and only if she likes the content creator (i.e., the realized $v_{ij} = V_j > 0$), and $V_j - p_j \geq 0$. Conditioned on the consumer doing so, the creator collects a total revenue of $p_j t_j^p$ given that each piece of content by j corresponds to one view.[8] For each unit of revenue from exclusive content, the platform takes a proportional commission r^p, leaving creator's share $1-r^p$.

Timing. The timing of the model is as follows:

1. The platform sets commissions r^η and r^p.
2. Creators simultaneously choose the amount of content t_j^η, t_j^p, and the access price p_j for exclusive content.
3. Consumers seek content recommendations and make viewing decisions.

3.1 Creators' problem

Consider the decision of a creator j with type-θ in the Stage 2 creator subgame. Provided that the creator sets $p_j \leq V_\theta$ so that the exclusive content is consumed, their maximization problem is

$$\max_{t_j^\eta, t_j^p, p_j} \pi_j = \max_{t_j^\eta, t_j^p, p_j} D_j(t_j^\eta)\left[(1-r^\eta)\eta t_j^\eta + (1-r^p)p_j t_j^p\right]\lambda_\theta - C\left(t_j^\eta + t_j^p\right),$$

where $C(\cdot)$ is an increasing convex function representing creators' cost of content production, and its derivative is denoted at $C'(\cdot)$. Trivially, $p_j = V_\theta$ as otherwise the creator can keep increasing the price without losing demand. The creator never sets $p_j > V_\theta$ as otherwise no consumer

6. A similar formulation of this matching process has been adopted in the literature of two-sided markets and payment card systems, see e.g. Rochet and Tirole (2003, 2006) and Bedre-Defolie and Calvano (2013), among others.

7. This does not have to take the form of additional content of the same type as the public content. In the context of Twitch, which is primarily a video game streaming website, this can take the form of add-on content such as specially designed chat emotes or events where the streamer plays games with viewers. Creators on Patreon will sometimes offer small chat sessions with supporters and some will even send "thank you" postcards. These all fit within the context of our model so long as consumers value this content and it requires effort on the part of creators.

8. Alternatively we can think of consumer utility being linear in the amount of content viewed, and $p_j t_j^p$ is the total price offered by the creator for accessing all of the exclusive content. Allowing for diminishing utility in the amount of content produced would not substantially change our results.

would watch exclusive content. The following convexity condition of guarantees quasi-concavity of the profit function.

Assumption 1. *For all $T > 0$, $\frac{C'(T)}{T} > 0$ is an increasing function.*

The standard first-order conditions deliver creator j's optimal choices of t_j^p and t_j^η. It is useful to note that the derivative of the recommendation probability can be written as

$$\frac{\partial D_j(t_j^\eta)}{\partial t_j^\eta} = \frac{D_j(t_j^\eta)\left(1 - D_j(t_j^\eta)\right)}{t_j^\eta}.$$

After imposing symmetry so that in the equilibrium all type-θ creators choose $(t_\theta^\eta, t_\theta^p)$ for $\theta = b, l$, the equilibrium content choices is characterized by the following simultaneous equation:

$$D_\theta(1 - r^p)V_\theta\lambda_\theta = C'\left(t_\theta^\eta + t_\theta^p\right) \tag{2}$$

and

$$\left(D_\theta\left(1 - D_\theta\right) + D_\theta\right)(1 - r^\eta)\eta\lambda_\theta + \frac{D_\theta\left(1 - D_\theta\right)}{t_\theta^\eta}(1 - r^p)V_\theta t_\theta^p\lambda_\theta = C'\left(t_\theta^\eta + t_\theta^p\right), \tag{3}$$

where

$$D_\theta = \frac{t_\theta^\eta W(\lambda_\theta)}{N_b t_b^\eta W(\lambda_b) + N_l t_l^\eta W(\lambda_l)}$$

is the the probability of a given type-θ creator being recommended in the equilibrium. These equilibrium conditions reflect the standard trade-off between marginal revenue from additional content and the marginal cost of providing content.

To rephrase the equilibrium in a more tractable fashion, we adopt the aggregative games approach similar to Acemoglu and Jensen (2013) and Anderson and Peitz (2020). To proceed, we rephrase each type-θ creator's choice as choosing their total content $T_\theta = t_\theta^p + t_\theta^\eta$ and the proportion of their total production devoted to public content, $\alpha_\theta \equiv \frac{t_\theta^\eta}{t_\theta^p + t_\theta^\eta}$. Then, denote the aggregate probability-weighted public content by

$$A = N_b W(\lambda_b)\alpha_b T_b + N_l W(\lambda_l)\alpha_l T_l. \tag{4}$$

The "aggregate" A can be interpreted as a proxy for how competitive the recommendation process is. The higher A is, the more effort all creators are putting in trying to get views, so the more effort each creator has to exert in order to get the same probability of being recommended.

The aggregative games approach reformulate the equilibrium choices of each creator in terms of the inclusive best reply functions, that is, "actions" that bring the aggregate probability-weighted public content to A.[9] After some simplifications, first-order conditions (2) and (3) can be rephrased in terms of A as follows:

$$\frac{C'\left(T_\theta\right)}{T_\theta} = \frac{\lambda_\theta W(\lambda_\theta)V_\theta(1 - r^p)}{A}\psi_\theta\left(T_\theta\right), \tag{5}$$

9. Note that this differs from the standard way to define best replies as functions of the actions of other players.

and $\alpha_\theta = \psi_\theta(T_\theta)$, where $\psi_\theta(.) \in [0,1]$ is a decreasing function defined by

$$\psi_\theta(T) \equiv \frac{(1-r^p)V_\theta\lambda_\theta - C'(T)}{2(1-r^p)V_\theta\lambda_\theta - C'(T)}\left[\frac{V_\theta(1-r^p)}{V_\theta(1-r^p)-\eta(1-r^\eta)}\right]. \tag{6}$$

Then, for each given A, a type-θ creator's inclusive best reply total content, $\mathbf{T}_\theta(A)$, is the solution T_θ that solves Equation (5); while the inclusive best reply proportion of public content is $\alpha = \psi_\theta(\mathbf{T}_\theta(A))$. Then, we can pin down the equilibrium aggregate probability-weighted public content, A^*, by substituting the inclusive best replies of every creator into Equation (4) and find the fixed point.

To ensure the uniqueness of the equilibrium, we impose the following assumption:[10]

Assumption 2. *For all $T > 0$, $T\psi_\theta(T)$ is increasing in T for $\theta = l, b$.*

Then, our first proposition below describes the equilibrium of Stage 2.

Proposition 1. *In the equilibrium of the Stage 2 creator subgame, each type-θ creator chooses $T_\theta = \mathbf{T}_\theta(A)$ and $a_\theta = \psi_\theta(T_\theta)$, where A^* is implicitly given by fixed-point condition*

$$A^* = \sum\nolimits_{\theta=l,b} N_\theta W(\lambda_\theta)\mathbf{T}_\theta(A^*)\psi_\theta(\mathbf{T}_\theta(A^*)). \tag{7}$$

The equilibrium is unique if Assumption 2 holds.

Proof. We prove uniqueness here as doing so is illustrative of the relationship between T and A. The remainder of the proof is relegated to the appendix. Using Assumption 1 and Equation (5), it is immediately clear that $\frac{d\mathbf{T}_\theta}{dA} < 0$ by the implicit function theorem. This then implies

$$\frac{d\psi_\theta}{dA} = \frac{\partial\psi_\theta}{\partial T}\frac{d\mathbf{T}_\theta}{dA} > 0.$$

Therefore, creators choose a lower total time investment and shift more time towards public content when the rival creators become more "aggressive". In other words, we have strategic complements in the sense that the amount of public content a creator produces increases with the amount of public content other creators produce. Second, Equation (7) will have a unique solution (and so the equilibrium will be unique) if the right hand side of Equation (7) is decreasing in A. This requires us to show

$$\frac{d}{dA}\psi_\theta\mathbf{T}_\theta = \left(\psi_\theta + \mathbf{T}_\theta\frac{\partial\psi_\theta}{\partial T}\right)\frac{d\mathbf{T}_\theta}{dA} < 0,$$

which is implied by the condition of $T\psi_\theta(T)$ being increasing. ■

3.1.1 Effects of fees on creators' behaviour

In this section, we analyze how the platform commission affect the equilibrium of the creator subgame. To facilitate the subsequent expositions, it is useful to define the extent of asymmetry

10. A sufficient condition is $C''(T) < 1/T$.

among the two types of creators:

$$\gamma = \max\left\{\frac{\lambda_b - \lambda_l}{\lambda_b}, \frac{V_l - V_b}{V_l}\right\} \in (0,1)$$

A small γ indicates that creators are not too asymmetric, and vice-versa. We first consider the public content commission, r^η.

Lemma 1. *(Effects of public content commission)*

- *A^*, α_l, and α_b are decreasing in r^η.*
- *T_b and D_b are decreasing in r^η while T_l and D_l are increasing.*
- *If η is small enough then the magnitude of these changes approaches zero for all γ.*
- *If γ is small enough then D_θ and T_θ do not change with r^η for $\theta = l, b$.*

Intuitively, from the inclusive best-response functions (5) and (6), notice that if we fix the aggregate A^* then $\frac{\partial \mathbf{T}_\theta(A^*)}{\partial r^\eta} < 0$ and $\frac{\partial \alpha}{\partial r^\eta} = \frac{\partial \psi_\theta(\mathbf{T}_\theta(A^*))}{\partial r^\eta} < 0$. This implies that if we hold the behavior of all other creators constant, then an increase in r^η induces a creator to reduce their total time investment (T) and the proportion of time allocated for public content (α). This is intuitive given a higher fee reduces the profitability of public content creation. Therefore, A^*, which measures the total effort all creators are putting in trying to get views, must decrease in the equilibrium. However, the reduced competitiveness of the rival creators raises some creators' incentive to raise T given $\frac{d\mathbf{T}_\theta}{dA} < 0$. Nonetheless, for the equilibrium α, we show that this reduced competition effect never dominates the effect of the reduced profitability, so that α_l and α_b are decreasing in r^η. As r^η increases, the opportunity cost of fighting for market share relative to producing public content increases because there is less revenue from the public content. This opportunity cost increases faster for the b creators because they get more revenue from public content than the l creators. Therefore the l creators' market share increases despite the fact that α_l is decreasing. However, if γ is small enough then the reduced competition effect exactly offsets the effect of the reduced profitability, so that T_l and T_b are independent of r^η.

We next consider the exclusive content commission, r^p.

Lemma 2. *(Effects of exclusive content commission) Suppose $r^\eta \to 1$ or η is small enough.*

- *A^* is decreasing in r^p.*
- *If γ is not too large, then T_b and T_l are decreasing in r^p.*
- *$\frac{dD_\theta}{dr^p}$ has the opposite sign as $\frac{d\alpha_\theta}{dr^p}$ for $\theta = l, b$. Moreover, if γ is small enough then D_θ and α_θ do not change with r^η*

We show later that $r^\eta \to 1$ is an appropriate assumption given η relatively small. Again, from the inclusive best-response function (5), notice that if we fix the aggregate A^* then $\frac{\partial \mathbf{T}_\theta(A^*)}{\partial r^p} < 0$. This again reflects the idea that an increase in r^p reduces profitability, thus induces creators to reduce their total time investment (T). At the same time however, the reduced competitiveness of the rival creators (as reflected by the decrease in A^*) raises some creators' incentive to raise T given $\frac{d\mathbf{T}_\theta}{dA} < 0$. This reflects the possibility that one of the two creator types decreases time investment so much that the other creator type has a large increase in recommendation probability and hence increases their time investment. Nonetheless, provided creators are not too asymmetric, this reduced competition effect never dominates the effect of the reduced profitability, so that T_b and T_l are decreasing in r^p. Assuming cubic cost functions, our numerical simulation suggests that this result generally survives asymmetry up to $\gamma = 0.7$ over a wide range of parameters.

3.2 Monopoly platform's pricing decision

Given the equilibrium of the creator subgame described in Proposition 1, the monopoly platform's problem is

$$\max_{r^\eta, r^p} \Pi^{hybrid} = \max_{r^\eta, r^p} \sum\nolimits_{\theta=l,b} \left(\alpha_\theta \eta r^\eta + (1-\alpha_\theta) V_\theta r^p\right) T_\theta \lambda_\theta D_\theta N_\theta.$$

Let

$$m_\theta = \alpha_\theta \eta r^\eta + (1-\alpha_\theta) V_\theta r^p$$

be the expected margin that the platform earns from each unit of time investment by a type-θ creator that is consumed.

Consider an increase in public content commission r^η.

Proposition 2. *If η is small relative to V_b and γ is sufficiently small, then in the hybrid monopolist model $r^\eta = 1$ and $r^p \in (0,1)$.*

To understand this result, note that the profit derivative $\frac{d\Pi^{hybrid}}{dr^\eta}$ is

$$\sum\nolimits_{\theta=l,b} \left(\alpha_\theta \eta + \frac{d\alpha_\theta}{dr^\eta}(\eta r^\eta - V_\theta r^p)\right) T_\theta \lambda_\theta D_\theta N_\theta + \sum\nolimits_{\theta=l,b} m_\theta T_\theta \lambda_\theta N_\theta \frac{dD_\theta}{dr^\eta} + \sum\nolimits_{\theta=l,b} m_\theta \frac{dT_\theta}{dr^\eta} \lambda_\theta D_\theta N_\theta. \tag{8}$$

The first term in Equation (8) indicates how r^η affects the platform's margin: it raises platform's revenue from the public content, but at the same time shifts content production from public to exclusive content. The shift in content creation pattern increases platform's profit when η is small relative to $\min\{V_b, V_l\} = V_b$, given that it implies that the revenue per exclusive content view is higher than for public content.

The second term in (8) reflects the fact that r^η affects the relative recommendation probability between the two types of creators, thus affecting the composition of the platform's revenue. The third term indicates that r^η decreases the total time the creators spend on producing content, T_θ, given the reduced profit. Remarkably, when γ is not too large, according to Lemma 1 $\frac{dD_\theta}{dr^\eta} = \frac{dT_\theta}{dr^\eta} = 0$, so that the last two effects indicated in Equation (8) vanish. Intuitively $\frac{dD_\theta}{dr^\eta} = 0$ because the composition effect is irrelevant when γ is small. $\frac{dT_\theta}{dr^\eta} = 0$ reflects the fact that discovery acts like a necessary input for creators. They receive no revenue if no one watches their content. Therefore their production of public content is fairly inelastic in r^η.

Because discovery is a necessary input for creators and the revenue from advertising is lower than for exclusive content, the production of exclusive content is much more responsive to the platform's commission than public content. Therefore r^p is the main determinant of T_θ for all creators. The platform's optimal choice of r^p solves the standard first-order condition of $\frac{d\Pi^{hybrid}}{dr^p} = 0$ (after subtituting for $r^\eta = 1$):

$$\sum\nolimits_{\theta=l,b} \left((1-\alpha_\theta) V_\theta + \frac{d\alpha_\theta}{dr^p} (\eta - V_\theta r^p) \right) T_\theta \lambda_\theta D_\theta N_\theta + \sum\nolimits_{\theta=l,b} m_\theta T_\theta \lambda_\theta N_\theta \frac{dD_\theta}{dr^p} = -\sum\nolimits_{\theta=l,b} m_\theta \frac{dT_\theta}{dr^p} \lambda_\theta D_\theta N_\theta.$$

When γ is small, $\frac{d\alpha_\theta}{dr^p} \to 0$ and $\frac{dD_\theta}{dr^p} \to 0$ by Lemma 2 so the condition simplifies to

$$\sum\nolimits_{\theta=l,b} (1-\alpha_\theta) V_\theta T_\theta \lambda_\theta D_\theta N_\theta = -\sum\nolimits_{\theta=l,b} m_\theta \frac{dT_\theta}{dr^p} \lambda_\theta D_\theta N_\theta. \tag{9}$$

Essentially, the platform trades-off between extracting more value from creators and reducing their content output.

4 Asymmetric duopoly platforms

The second market structure we are considering in this work is an asymmetric duopoly with a pure discovery platform and a pure membership platform competing for content creators' time investment. Each platform faces its own profit maximization problem, but the rest of the model is unchanged apart from the timing:

Timing. The timing of the model is as follows:

1. The platforms set commissions r^η and r^p.
2. Creators simultaneously choose the amount of content t_j^η, t_j^p, and the access price p_j for the exclusive content.
3. Consumers seek content recommendations and make viewing decisions.

We note that for each given r^η and r^p, the analysis of Stage 2 and Stage 3 is the same as in Section 3.1 and so the equilibrium characterization in Proposition 1 still applies. In Stage 1, the pure discovery platform's problem is then

$$\max_{r^\eta} \Pi^{disc} = \max_{r^\eta} \sum\nolimits_{\theta=l,b} \eta r^\eta \alpha_\theta T_\theta \lambda_\theta D_\theta.$$

Analogous to Proposition 2, we find that the dominant strategy of the pure discovery platform is to set the highest possible r^η:

Proposition 3. *If γ is not too large then in the asymmetric duopoly model $r^\eta = 1$ and $r^p \in (0,1)$.*

The intuition here is similar to Proposition 2, note that the profit derivative $\frac{d\Pi^{disc}}{dr^\eta}$ is

$$\sum\nolimits_{\theta=l,b} \left(\alpha_\theta \eta + \frac{d\alpha_\theta}{dr^\eta} \eta r^\eta \right) T_\theta \lambda_\theta D_\theta N_\theta + \sum\nolimits_{\theta=l,b} \alpha_\theta \eta r^\eta T_\theta \lambda_\theta N_\theta \frac{dD_\theta}{dr^\eta}$$
$$+ \sum\nolimits_{\theta=l,b} \alpha_\theta \eta r^\eta \frac{dT_\theta}{dr^\eta} \lambda_\theta D_\theta N_\theta.$$

Notice that $\alpha_\theta \eta r^\eta < m_\theta$, reflecting that the duopolist discovery platform does not benefit from increased production of exclusive content. Nonetheless, given that $\frac{dD_\theta}{dr^\eta} = \frac{dT_\theta}{dr^\eta} = 0$ when γ is small, $\frac{d\Pi^{disc}}{dr^\eta}$ has the same sign as $\alpha_\theta \eta + \frac{d\alpha_\theta}{dr^\eta}\eta r^\eta > 0$, so $r^\eta = 1$. This again reflects the logic that the inelasticity of public content provision to the platform's commission (because of its status as an essential input) is sufficiently strong.

Meanwhile, the pure membership platform's problem is

$$\max_{r^p} \Pi^{mem} = \max_{r^p} \sum\nolimits_{\theta=l,b} V_\theta r^p (1-\alpha_\theta) T_\theta \lambda_\theta D_\theta.$$

The platform's optimal choice of r^p solves the standard first-order condition of $\frac{d\Pi^{mem}}{dr^p} = 0$ (after subtituting for $r^\eta = 1$):

$$\begin{aligned} &\sum\nolimits_{\theta=l,b} \left((1-\alpha_\theta) V_\theta - \frac{d\alpha_\theta}{dr^p} V_\theta r^p \right) T_\theta \lambda_\theta D_\theta N_\theta + \sum\nolimits_{\theta=l,b} (1-\alpha_\theta) V_\theta r^p T_\theta \lambda_\theta N_\theta \frac{dD_\theta}{dr^p} \\ = \quad & -\sum\nolimits_{\theta=l,b} (1-\alpha_\theta) V_\theta r^p \frac{dT_\theta}{dr^p} \lambda_\theta D_\theta N_\theta. \end{aligned}$$

When γ is small, $\frac{d\alpha_\theta}{dr^p} \to 0$ and $\frac{dD_\theta}{dr^p} \to 0$ by Lemma 2 so the condition simplifies to

$$\sum\nolimits_{\theta=l,b} (1-\alpha_\theta) V_\theta T_\theta \lambda_\theta D_\theta N_\theta = -\sum\nolimits_{\theta=l,b} (1-\alpha_\theta) V_\theta r^p \frac{dT_\theta}{dr^p} \lambda_\theta D_\theta N_\theta. \tag{10}$$

This is broadly similar to Equation (9), except that the pure membership platform does not care about advertising revenue. Compared to the monopoly platform, the marginal cost of increasing the exclusive content commission is lower for the membership platform. This comparison leads directly to Proposition 4

Proposition 4. *(Pricing comparison) If η is small relative to V_b, and γ is not too large, then the hybrid monopolist will set r^p lower than a pure membership platform in a duopoly. Under the integrated market structure there is more production of both public and exclusive content than in the duopoly setting.*

This result is reminiscent of the results in the classic literature on multi-product monopolist. The monopoly pricing takes into account any consumption complementary across its multiple products, so it tends to charge lower (hence more efficient) prices compared to competing duopoly. Our main distinction is that the "complementarity" in our setup is due to the production behavior of content creators rather than any consumption synergy from the consumer side. In our model, a higher exclusive content fee reduces the amount of public content because the fee lowers creators' per-view profits, meaning that they have a weaker incentive to compete for viewers through the production of public content.

Numerical simulations in Figure 1 below show that Proposition 4 survives even when the extent of asymmetry between creators, γ, is large. In Figure 1, we impose cubic cost function $C(T) = \frac{1}{3}T^3$, $\eta = 0.1$, $V_l = 5$ and $\lambda_b = 0.5$, and plot the equilibrium r^p and r^η in by the hybrid monopolist and the asymmetric duopolists over $\gamma \in [0, 0.8]$. The first panel shows that the pure membership platform always sets a higher r^p for all γ than the monopolist. The second panel shows that both the hybrid monopolist and the pure discovery platforms set the highest possible $r^\eta = 1$ and slightly below 1 when γ is very large. These observations continue to hold for lower values of η and higher values of V_l. In practice, Online CPM (revenue per 1,000 ad views) is often below \$10, which would give $\eta = \$0.01$ while a typical monthly contribution on Patreon is $V_l = \$5$, which suggests that these numerical simulations are highly conservative and that the result in Proposition 4 is quite general.

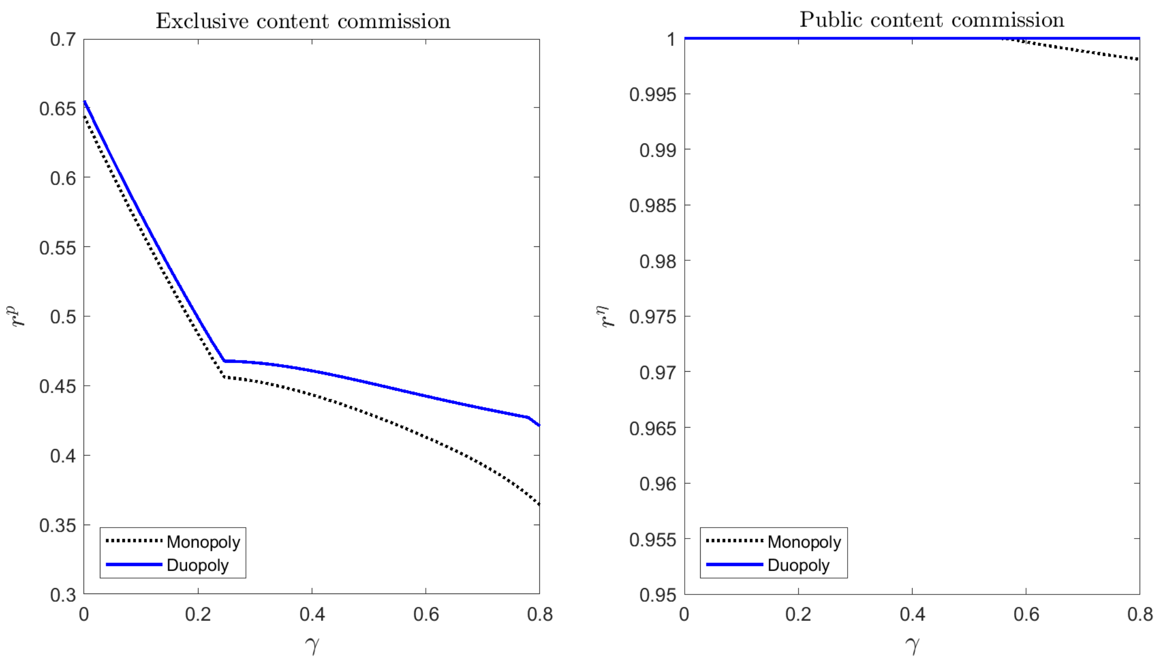

Figure 1: The equilibrium commissions in monopoly and asymmetric duopoly. We assume $N_b = N_l = 5$, $\lambda_l = (1-\gamma)\lambda_b$, $V_b = (1-\gamma)V_l$, and linear weighting function $W(\lambda) = \lambda$.

5 Design of the recommendation algorithm

In this section, we endogenize the platform design of the recommendation algorithm, and analyze how the platform market structure affects the equilibrium design choices. Specifically, we assume that the platform can choose the weightage function $W(.)$ used in (1). Given our assumption of two creator types it is without loss of generality to set $W(\lambda_l) = 1$ and define $W \equiv W(\lambda_b) > 0$. We do not impose any restriction on the range of feasible W so that the platform is unconstrained in its design problem, except that it needs to take into account the equilibrium behaviours of the content creators. We discuss below how imposing restrictions on the range of feasible W affects our results.

5.1 Effects of design on creators' behaviour

Changing the platform decision variable does not change the creators' problem, so equilibrium of the creator subgame is still determined by Proposition 1. We have the following result on the effect on w on the Stage-2 equilibrium:

Lemma 3. *(Effects of recommendation weight for broad creators, W)*

- A^*, T_b, $\alpha_b T_b$, *and* D_b *are increasing in* W.
- T_l, $\alpha_l T_l$, *and* D_l *are decreasing in* W.

Intuitively, shifting the recommendation weight in favor of b creators increases that the marginal benefit they receive from creating public content. This increase the total public content production of the b creator while simultaneously decreasing that for the l creators. Consequently, in the equilibrium the probability of a given type-b creator being recommended in the equilibrium, D_b, becomes higher.

5.2 Hybrid monopoly platform

To focus on the design problem, in what follows we assume that the platform's commission is held constant (so that r^p and r^η are exogenous).[11] Consistent with Section 3.2, we exogenously impose η is small such that $\eta < r^p V_b < r^p V_l$. To facilitate clear exposition, we also restrict our attention to a model with cost function: $C(T) = \frac{c}{3}T^3$, where $c > 0$ is the cost parameter.[12]

The monopoly platform chooses W to maximizes its profit

$$\sum\nolimits_{\theta=l,b} \left(\alpha_\theta \eta r^\eta + (1-\alpha_\theta) V_\theta r^p\right) T_\theta \lambda_\theta D_\theta N_\theta.$$

One important technical implication from Lemma 3 is that D_b is monotone increasing in W. Therefore, without loss of generality, we can reframe the platform design decision as directly choosing the equilibrium $D_b \in [0, 1/N_b]$, where $D_b = 1/N_b$ corresponds to a maximal weight on recommending broad creators and $D_b = 0$ corresponds to the minimal weight. The recommendation probability of a given type-l creator can be obtained through $N_l D_l = 1 - N_b D_b$. Then, from Equation (5) and Equation (6), we can pin down the relationship between the equilibrium[13] $(\alpha_\theta, T_\theta)$ and D_θ as: $\alpha_\theta = \frac{1-D_\theta}{2-D_\theta}$ and $T_\theta = (C')^{-1}(D_\theta(1-r^p)V_\theta\lambda_\theta)$.

Given the reformulation, the platform's problem becomes:

$$\max_{D_b} \Pi^{hybrid} = \max_{D_b \in [0,1/N_b]} \left\{ \begin{array}{c} \left(\alpha_b \eta r^\eta + (1-\alpha_b) V_b r^p\right) T_b \lambda_b D_b N_b \\ + \left(\alpha_l \eta r^\eta + (1-\alpha_l) V_l r^p\right) T_l \lambda_l \left(1 - D_b N_b\right) \end{array} \right\}.$$

Proposition 5 characterizes the hybrid monopolist's optimal design.

Proposition 5. *The hybrid monopolist's profit is convex in* D_b. *There exists a threshold* $\bar{V}^{hybrid} \in [0, V_l]$ *such that the hybrid monopolist optimally chooses* $D_b = 1/N_b$ *if* $V_b \geq \bar{V}^{hybrid}$

11. This assumption of exogenous price means that the membership platform is essentially silent in the duopolist model apart from giving creators an alternative form of monetization. While we could appeal to the envelope theorem in the case of the hybrid monopolist and the problem would essentially remain unchanged, solving the duopoly problem would become much less tractable. Assuming that weights are set in the same stage of the game as prices seems unrealistic, so we would need to account for the impact of the discovery platform's weighting decision on the membership platform's prices (or vice versa). Additionally, the envelope theorem would not reduce complexity for comparative statics.

12. Our results carry through with a general cost function $C(\cdot)$ provided that C' is log-concave and that N_b and N_l are sufficiently large. Details are available upon request.

13. Note that $(\alpha_\theta, T_\theta)$ are not referring to the best-inclusive functions here given the reformulation.

and $D_b = 0$ otherwise. Moreover, the threshold $\bar{V}^{hybrid}$ is increasing in N_b λ_l, and V_l, and it is decreasing in λ_b and N_l.

An increase in D_b will increase the total content production of the b creators, as well as increasing the proportion of content they devote to exclusive content, while simultaneously decreasing both for the l creators. The platform's profits are quasi-convex in this behavior because the platform is benefiting from the increased exclusive content production. The platform's profit is convex in this behavior because the platform is benefiting from the increased exclusive content production given η is small.

Given the convexity, the maximization problem boils down to comparing between the extreme points. Assigning the maximal weight on recommending broad creators is optimal for the monopoly platform if and only if $\Pi^{hybrid}_{D_b=1/N_b} > \Pi^{hybrid}_{D_l=1/N_l}$ (note $D_l = 1/N_l$ is equivalent to $D_b = 0$). Proposition 5 indicates that the monopoly platform is more likely to focus on recommending the broad creators when V_b and λ_b are large relative to V_l and λ_l, or when N_b is small relative to N_l.

To understand this result, we show in the proof that $\Pi^{hybrid}_{D_b=1/N_b} > \Pi^{hybrid}_{D_l=1/N_l}$ holds if and only if $Z_b > Z_l$, where

$$Z_\theta = \left(\left(\frac{N_\theta - 1}{2N_\theta - 1}\right)\lambda_\theta + \left(\frac{N_\theta}{2N_\theta - 1}\right)\left(\frac{r^p}{r^\eta \eta}\right)\lambda_\theta V_\theta\right)(C')^{-1}\left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta}\right) \tag{11}$$

is proportional to the platform's profit when focusing on recommending type-θ creators. Obviously, Z_θ is increasing in λ_θ and V_θ given that these imply: (i) a higher content consumption probability conditioned on type-θ being recommended; (ii) more value generated from the exclusive content, and (iii) a higher total time investment and proportion of time devoted to exclusive content by these creators. One counter-intuitive observation is that the profitability measure Z_θ is decreasing in N_θ. The main reason is that a higher N_θ intensifies competition between type-θ creators if they are assigned positive weight. More competition implies a lower incentive for time investment by each individual creator and hence a lower profit for the platform.

Consider what happens when we impose restrictions on the set of feasible design so that $D_b \in [\underline{D}, \bar{D}]$ for some arbitrary lower bound and upper bound $\underline{D} > 0$ and $\bar{D} < 1/N_b$. The analysis above remains the same except that we will be comparing platform's profit between end points $D_b = \underline{D}$ and $D_l = \bar{D}$. Given that profit function is convex, a tighter upper bound $\bar{D}$ simply shifts the profit comparison in favor of assigning less weight on broad creators. Likewise, a tighter lower bound $\underline{D}$ simply shifts the profit comparison in favor of assigning more weight on broad creators.

5.3 Asymmetric duopoly platforms

Using the same reformulation as in the previous section, the problem of a pure discovery platform in an asymmetric duopoly is

$$\max_{D_b} \Pi^{disc} = \max_{D_b \in [0,1/N_b]} \left\{ \begin{array}{c} \alpha_b \eta r^\eta T_b \lambda_b D_b N_b \\ +\alpha_l \eta r^\eta T_l \lambda_l (1 - D_b N_b) \end{array} \right\}.$$

Proposition 6 characterizes the hybrid monopolist's optimal design.

Proposition 6. *If $N_b > 3$ and $N_l > 3$, the pure discovery duopolist's profit is convex in D_b. There exists a threshold $\bar{V}^{disc} \in [0, V_l]$ such that the duopolist optimally chooses $D_b = 1/N_b$ if $V_b \geq \bar{V}^{disc}$ and $D_b = 0$ otherwise. Moreover, the threshold $\bar{V}^{disc}$ is increasing in N_b λ_l, and V_l, and it is decreasing in λ_b and N_l.*

The reasoning for convexity of the profit function is similar to Proposition 5. Then, comparing discovery platform's profit between the end points, we have $\Pi^{disc}_{D_b=1/N_b} > \Pi^{disc}_{D_l=1/N_l}$ if and only if $\tilde{Z}_b > \tilde{Z}_l$, where

$$\tilde{Z}_\theta = \left(\frac{N_\theta - 1}{2N_\theta - 1}\right) \lambda_\theta (C')^{-1} \left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta}\right). \tag{12}$$

Comparing this measurement to the corresponding term in the monopoly case in (11), we note that the relevant profit comparison is radically different between the pure discovery platform and the hybrid monopolist because the pure discovery platform does not benefit from creation of exclusive content. As such, the pure discovery platform's objective function is essentially to maximize the number of views (and thus places much more emphasis on λ_θ), while the monopolist takes into account the value generated from exclusive content and hence it takes V_θ into account as well).

Formally, if we compare Proposition 5 and Proposition 6, we obtain the following result:

Proposition 7. *Comparing the thresholds in Proposition 5 and Proposition 6, $\bar{V}^{hybrid} \geq \bar{V}^{disc}$ if and only if*[14]

$$\frac{N_l V_l}{N_l - 1} \geq \frac{N_b \bar{V}^{disc}}{N_b - 1}. \tag{13}$$

Proposition 7 has a few novel implications. First, recall from Proposition 6 that $\bar{V}^{disc} \leq V_l$, and so condition (13) holds, e.g., when $N_l = N_b$ (or more generally any $N_l \leq N_b$ given $\frac{N}{N-1}$ is decreasing). Therefore, $\bar{V}^{hybrid} \geq \bar{V}^{disc}$ is likely to hold, meaning that the range of parameter for the hybrid monopolist to choose $D_b = 1/N_b$ (i.e., when $V_b > \bar{V}^{hybrid}$) is smaller than the corresponding range for the pure discovery platform to choose $D_b = 1/N_b$. This suggests that the pure discovery platform tends to focus more on recommending broad content creators relative to the hybrid monopoly platform. This is consistent with the intuition discussed above.

5.4 Welfare

The exclusive content price p_j, and the commissions r^p and r^η are transfers, so we can write total welfare (total surplus) as

$$\Omega = N_b \left(D_b \lambda_b (V_b + \alpha_b \eta) T_b - C(T_b)\right) + N_l \left(D_l \lambda_l (V_l + \alpha_l \eta) T_l - C(T_l)\right)$$

Which we can use to find the following proposition

14. Condition (13) can be equivalently stated as $\frac{N_l V_l}{N_l - 1} \geq \frac{N_b \bar{V}_{hybrid}}{N_b - 1}$.

Proposition 8. *If $N_b > 3$ and $N_l > 3$, total welfare is convex in D_b. There exists a threshold $\bar{V}^{welfare} \in [0, V_l]$ such that the welfare maximization entails $D_b = 1/N_b$ if $V_b > \bar{V}^{welfare}$, and $D_b = 0$ otherwise. Moreover, the threshold $\bar{V}^{welfare}$ is increasing in V_l and λ_l, and decreasing in V_b and λ_b.*

Most of this proposition is fairly intuitive. Notice that we do not state how $\bar{V}^{welfare}$ changes with N_θ. This is because changes in N_θ have three effects on welfare:

1. Increased competition reduces total content production per creator, which decreases the total surplus.
2. $N_\theta C(T_\theta)$ decreases, the total amount of content produced per creator decreases even though the number of creators is increasing. Spreading out the content among more creators decreases the total cost because of the convexity of the cost function. This effect increases the total surplus.
3. Increased competition among creators decreases production of public content because competing for market share becomes less attractive, as compared to making the relatively more profitable exclusive content. Given that each public content view adds advertising revenue η to the total surplus, this effect decreases total surplus.[15]

If we compare Proposition 8 with Proposition 5 and Proposition 6, we obtain the following result:

Proposition 9. *Comparing the thresholds, $\bar{V}^{welfare} \geq \bar{V}^{disc}$ if and only if*

$$\left(\frac{2N_b - 1}{N_b - 1} V_b - \frac{2N_l - 1}{N_l - 1} V_l \right) \tilde{Z}_l \leq N_b C\left(T_b\right) - N_l C\left(T_l\right), \tag{14}$$

and $\bar{V}^{welfare} \geq \bar{V}^{hybrid}$ if and only if

$$\left(\frac{\frac{2N_b-1}{N_b-1} V_b + \eta}{1 + \frac{N_b}{N_b-1}\left(\frac{r^p}{r^\eta \eta}\right) V_b} - \frac{\frac{2N_l-1}{N_l-1} V_l + \eta}{1 + \frac{N_l}{N_l-1}\left(\frac{r^p}{r^\eta \eta}\right) V_l} \right) Z_l \leq N_b C\left(T_b\right) - N_l C\left(T_l\right), \tag{15}$$

where

$$N_\theta C\left(T_\theta\right) = N_\theta C\left(C'^{-1}\left(\frac{(1 - r^p)\lambda_\theta V_\theta}{N_\theta} \right) \right)$$

and both (14) and (15) are evaluated at $V_b = \bar{V}^{welfare}$.

To understand the comparison of cutoffs, first suppose that $N_b C\left(T_b\right) - N_l C\left(T_l\right) \approx 0$ (so that there is no distortion from the platform(s) failing to account for production costs), and $N_b = N_l = \frac{N}{2}$. Then Equation (14) always holds given $\bar{V}^{welfare} \leq V_l$, meaning that $\bar{V}^{welfare} \geq \bar{V}^{disc}$.

15. Note that we do not include a nuisance cost of advertising on consumers. If we were to include a nuisance cost and it were greater than η, all of our results would still go through, but this third effect would increase total surplus. That said, the tradeoff of reduced content production per creator vs. reduced total production cost still means the effect of N_θ on welfare is ambiguous.

This means that the discovery platform tends to over-emphasize on b creators. Intuitively, total surplus takes into account the higher value generated by l creators, while the discovery platform does not.

Similarly, Equation (15) becomes

$$\frac{\frac{N-1}{N/2-1}\bar{V}^{welfare}+\eta}{1+\frac{N/2}{N/2-1}\left(\frac{r^p}{r^\eta\eta}\right)\bar{V}^{welfare}}\leq\frac{\frac{N-1}{N/2-1}V_l+\eta}{1+\frac{N/2}{N/2-1}\left(\frac{r^p}{r^\eta\eta}\right)V_l}.$$

Given $\bar{V}^{welfare}\leq V_l$, the condition holds if and only if

$$\frac{d}{dV}\left(\frac{\frac{N-1}{N/2-1}V+\eta}{1+\frac{N/2}{N/2-1}\left(\frac{r^p}{r^\eta\eta}\right)V}\right)=\frac{\frac{N-1}{N/2-1}-\eta\left(\frac{N/2}{N/2-1}\left(\frac{r^p}{r^\eta\eta}\right)\right)}{\left(1+\frac{N/2}{N/2-1}\left(\frac{r^p}{r^\eta\eta}\right)V_b\right)^2}>0$$

or

$$N-1-\frac{N}{2}\left(\frac{r^p}{r^\eta}\right)>0 \tag{16}$$

The results depend on $\frac{r^p}{r^\eta}$. For instance, if $r^p\approx 0$ then condition (16) always holds, so $\bar{V}^{welfare}\geq\bar{V}^{hybrid}$. This is because the hybrid monopolist earns nothing from exclusive content and consequently over-emphasizes the b creators.[16]

Following Proposition 2, if we suppose that $r^\eta\approx 1$, which is consistent with Proposition 2, then (16) holds for all $N>2$, suggesting that $\bar{V}^{welfare}\geq\bar{V}^{hybrid}$ is more likely to occur. Together with Proposition 7, this implies $\bar{V}^{welfare}\geq\bar{V}^{hybrid}\geq\bar{V}^{disc}$, meaning that the range of parameters where the hybrid monopolist chooses the welfare-maximizing design is larger, than that where the asymmetric duopolist's design is welfare-optimal. This reflects the fact that the hybrid monopolist obtains revenue from both types of contents, so that its objective function is more closely aligned with the welfare function.

In both market structures, once we take into account the cost effect then a the condition becomes more likely to hold if $N_bC\left(T_b\right)-N_lC\left(T_l\right)$ increases. Intuitively, when the overall cost incurred by the broad creators is higher, the welfare maximization will shift in favor of limited creators.

6 Conclusion

We develop a tractable model a of platform-intermediated content creation market, whereby heterogenous content creators endogenously divide their time between making advertising funded public content and subscription/membership funded exclusive content. A pure discovery platform facilitates audiences' discovery of creators' content, a pure membership platform serves as a medium enabling creators to profit from providing exclusive content, and a hybrid monopoly platform combines both functionalities.

By comparing a hybrid monopolist platform to an asymmetric duopoly (with a pure membership platform and a pure discovery platform), our main results show that the monopolist tends

16. To the other extreme, if $r^\eta\approx 0$, we have the opposite result of $\bar{V}_{welfare}\leq\bar{V}_{hybrid}$. In this case, hybrid monopolist earns revenue primarily through exclusive content and over-emphasizes the l creators.

to make more welfare-efficient choices in terms of (i) the level of its fees and (ii) the design of its recommendation algorithm. The key intuition is that endogenous creation decisions by content creators lead to a complementary relationship between the two types of content, and that the hybrid monopolist partially internalizes this complementarity because it obtains revenue from both types of content.

A Proofs

A.1 Proof of Proposition 1

Consider the maximization problem of creator j.

$$\frac{d\pi}{dt_j^p} = \frac{t_j^\eta W(\lambda_j)}{A}(1-r^p)V_j\lambda_j - C'\left(t_j^\eta + t_j^p\right)$$

Meanwhile, we know

$$\frac{\partial D(t_j^\eta)}{\partial t_j^\eta} = \frac{W(\lambda_j)(A - t_j^\eta W(\lambda_j))}{A^2}$$

so

$$\begin{aligned}\frac{d\pi}{dt_j^\eta} &= \left(\frac{\partial D(t_j^\eta)}{\partial t_j^\eta}t_j^\eta + D(t_j^\eta)\right)(1-r^\eta)\eta\lambda_j + \frac{\partial D(t_j^\eta)}{\partial t_j^\eta}(1-r^p)V_j t_j^p\lambda_j - C'\left(t_j^\eta + t_j^p\right) \\ &= \left(\frac{2A - t_j^\eta W(\lambda_j)}{A^2}\right)(1-r^\eta)\eta\lambda_j W(\lambda_j)t_j^\eta + \frac{D_j(t_j^\eta)\left(1 - D_j(t_j^\eta)\right)}{t_j^\eta}(1-r^p)V_j t_j^p\lambda_j - C'\left(t_j^\eta + t_j^p\right).\end{aligned}$$

We can combine these two first-order conditions to yield

$$\frac{t_j^\eta}{t_j^p + t_j^\eta} = \frac{A - t_j^\eta W(\lambda_j)}{2A - t_j^\eta W(\lambda_j)}\left[\frac{V_j(1-r^p)}{V_j(1-r^p) - \eta(1-r^\eta)}\right].$$

As in the symmetric case, we need $V_j(1-r^p)$ higher enough to obtain interior solutions. At this point the problem becomes easier if we rephrase the creators choice as choosing total time investment T_j and the proportion of investment $\alpha_j \equiv \frac{t_j^\eta}{t_j^p + t_j^\eta}$, so that the first-order conditions become

$$\frac{\alpha\lambda_j W(\lambda_j)V_j(1-r^p)}{A} = \frac{C'(T)}{T} \tag{17}$$

and

$$\alpha = \frac{A - \alpha T W(\lambda_j)}{2A - \alpha T W(\lambda_j)}\left[\frac{V_j(1-r^p)}{V_j(1-r^p) - \eta(1-r^\eta)}\right]. \tag{18}$$

We can combine both equations to substitute away α to get Equation (5). After imposing symmetry so that in the equilibrium all type-θ creators choose $(t_\theta^\eta, t_\theta^p)$ for $\theta = b, l$, we recover the characterization stated in the main text. The remaining of the proof is stated in the main text.■

A.2 Proof of Lemma 1

From the definition of A^* in Equation (7), by implicit function theorem, $\frac{dA^*}{dr^\eta}$ has the same sign as $\frac{\partial\psi_\theta \mathbf{T}_\theta}{\partial r^\eta}$. Notice

$$\begin{aligned}
\frac{\partial\psi_\theta \mathbf{T}_\theta}{\partial r^\eta} &= \psi_\theta \frac{\partial \mathbf{T}_\theta}{\partial r^\eta} + \left(\frac{\partial\psi_\theta}{\partial r^\eta} + \frac{\partial\psi_\theta}{\partial T}\frac{\partial \mathbf{T}_\theta}{\partial r^\eta}\right)\mathbf{T}_\theta \\
&= \frac{\partial\psi_\theta}{\partial r^\eta}\mathbf{T}_\theta + \left(\psi_\theta + \mathbf{T}_\theta\frac{\partial\psi_\theta}{\partial T}\right)\frac{\partial \mathbf{T}_\theta}{\partial r^\eta} \\
&= \frac{\partial\psi_\theta}{\partial r^\eta}\left[\mathbf{T}_\theta + \left(\psi_\theta + \mathbf{T}_\theta\frac{\partial\psi_\theta}{\partial T}\right)X\right],
\end{aligned}$$

where the last line follows from (5), which gives $\frac{\partial \mathbf{T}_\theta}{\partial r^\eta} = X\frac{\partial\psi_\theta}{\partial r^\eta} < 0$, where

$$X = \frac{\frac{\lambda_\theta W(\lambda_\theta)V_\theta(1-r^p)}{A}}{\frac{\partial C'(T)/T}{\partial T} - \frac{\lambda_\theta W(\lambda_\theta)V_\theta(1-r^p)}{A}\frac{\partial\psi_\theta}{\partial T}} > 0. \tag{19}$$

Recall Assumption 2 implies $\psi_\theta + \frac{\partial\psi}{\partial T}\mathbf{T}_\theta > 0$. Therefore, if $\frac{\partial\psi_\theta}{\partial r^\eta} < 0$ for both $\theta = l, b$ then we are done, and this is obviously true from (6). We conclude $\frac{dA^*}{dr^\eta} < 0$.

Next,

$$\begin{aligned}
\frac{d\alpha_\theta}{dr^\eta} &= \frac{\partial\psi_\theta}{\partial r^\eta} + \frac{\partial\psi_\theta}{\partial T}\left(\frac{\partial \mathbf{T}_\theta}{\partial r^\eta} + \frac{d\mathbf{T}_\theta}{dA}\frac{dA^*}{dr^\eta}\right). \\
&= \left(1 + \frac{\partial\psi_\theta}{\partial T}X\right)\frac{\partial\psi_\theta}{\partial r^\eta} + \frac{d\mathbf{T}_\theta}{dA}\frac{dA^*}{dr^\eta}
\end{aligned}$$

given that $\frac{\partial\psi_\theta}{\partial T}X < -1$ from Equation (19).

Next, notice that we can rearrange Equation (5) to get

$$D_\theta(1-r^p)V_\theta\lambda_\theta = C'(T_\theta) \tag{20}$$

If the time investments change asymmetrically in the same direction then this equation cannot be satisfied for both types as the LHS will have increased for one and decreased for the other, but the RHS will have changed in the same direction for both. If the D_θ were to remain unchanged, then the optimal choices of T_θ would be unchanged, but then α_θ would have changed, which would change the D_θ (i.e. we get a contradiction). So that the T_θ have to change, and must change in opposing directions for the two creator types. Now, take Equation (5) for both types and divide one by the other to get

$$\frac{\frac{C'(T_l)}{T_l}}{\frac{C'(T_b)}{T_b}} = \frac{W(\lambda_l)\lambda_l V_l}{W(\lambda_b)\lambda_b V_b}\frac{\frac{(1-r^p)V_l\lambda_l - C'(T_l)}{2(1-r^p)V_l\lambda_l - C'(T_l)}}{\frac{(1-r^p)V_b\lambda_b - C'(T_b)}{2(1-r^p)V_b\lambda_b - C'(T_b)}}\frac{V_l}{V_b}\left(\frac{V_b(1-r^p)-(1-r^\eta)\eta}{V_l(1-r^p)-(1-r^\eta)\eta}\right)$$

It is straightforward to establish that

$$\frac{\partial}{\partial r^\eta}\left(\frac{V_b(1-r^p)-(1-r^\eta)\eta}{V_l(1-r^p)-(1-r^\eta)\eta}\right)$$

has the same sign as $\eta(1-r^p)(V_l - V_b) > 0$, so this term will increase as r^η increases. Given that

$$\frac{\partial}{\partial T}\left(\frac{(1-r^p)V_\theta\lambda_\theta - C'(T_\theta)}{2(1-r^p)V_\theta\lambda_\theta - C'(T_\theta)}\right) = \frac{-(1-r^p)V_\theta\lambda_\theta C''}{(2(1-r^p)V_\theta\lambda_\theta - C'(T_\theta))^2} < 0$$

T_b must decrease and T_l increase in order to restore equality.

Next,

$$\begin{aligned} \frac{dT_\theta}{dr^\eta} &= \frac{\partial \mathbf{T}_\theta}{\partial r^\eta} + \frac{\partial \mathbf{T}_\theta}{\partial A}\frac{dA^*}{dr^\eta} \\ &= \left(\frac{\partial \psi_\theta}{\partial r^\eta} - \frac{dA^*}{dr^\eta}\frac{\psi_\theta}{A^*}\right)X. \end{aligned}$$

where we used $\frac{\partial \mathbf{T}_\theta}{\partial r^\eta} = X\frac{\partial \psi_\theta}{\partial r^\eta}$ and $\frac{\partial \mathbf{T}_\theta}{\partial A} = -X\frac{\psi_\theta}{A^*}$ from Equation (5), and Equation (19). Then,

$$\begin{aligned} & \frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{A^*}\frac{dA^*}{dr^\eta} \\ = \; & \frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\frac{\psi_\theta}{A^*}\sum_{\theta=l,b} N_\theta W(\lambda_\theta)\frac{\partial \psi_\theta}{\partial r^\eta}\left[\mathbf{T}_\theta + \left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\right]}{1 + \sum_{\theta=l,b} N_\theta W(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}}. \end{aligned}$$

If γ is small enough, then $\psi_b \to \psi_l$ and $\frac{\partial \psi_b}{\partial r^\eta} \to \frac{\partial \psi_l}{\partial r^\eta}$ so that the last line becomes

$$\begin{aligned} &= \frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\frac{\psi_\theta}{A^*}NW(\lambda_\theta)\frac{\partial \psi_\theta}{\partial r^\eta}\left[\mathbf{T}_\theta + \left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\right]}{1 - NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}} \\ &= \frac{\partial \psi_\theta}{\partial r^\eta}\left(1 - \frac{1 + \frac{\psi_\theta}{A^*}NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X}{1 + NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}}\right) = 0. \end{aligned}$$

where the second equality uses $\psi_\theta NW(\lambda_\theta)\mathbf{T}_\theta = A^*$. So $\frac{dT_\theta}{dr^\eta} \to 0$ for both $\theta = l, b$. Moreover, if $\gamma \to 0$ then $D_\theta \to 1/N$, independent of r^η.

For the last statement, notice from the proofs above that $\frac{dA^*}{dr^\eta}$, $\frac{dT_\theta}{dr^\eta}$, and $\frac{d\alpha_\theta}{dr^\eta}$ are proportional to $\frac{\partial \psi_\theta}{\partial r^\eta}$. When $\eta \to 0$, then

$$\frac{\partial \psi_\theta}{\partial r^\eta} = \frac{-\psi_\theta \eta}{V_\theta(1-r^p) - \eta(1-r^\eta)} \to 0,$$

so that all the relevant derivatives becomes zero.■

A.3 Proof of Lemma 2

From the definition of A^* in Equation (7), by implicit function theorem, $\frac{dA^*}{dr^p}$ has the same sign as $\frac{\partial \psi_\theta \mathbf{T}_\theta}{\partial r^p}$. Notice that when $r^\eta \to 1$ or η is small enough, from (6),

$$\frac{\partial \psi_\theta}{\partial r^p} = \frac{-V_\theta\lambda_\theta C'(T_\theta)}{(2\lambda_\theta V_\theta(1-r^p) - C'(T_\theta))^2} < 0$$

and $\frac{\partial \mathbf{T}_\theta}{\partial r^p} = X\left(\frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{1-r^p}\right) < 0$ from (5), so $\frac{dA^*}{dr^p} \leq 0$. Next, similar to Lemma 1, we write

$$\begin{aligned}
\frac{dT_\theta}{dr^p} &= \frac{\partial \mathbf{T}_\theta}{\partial r^p} + \frac{\partial \mathbf{T}_\theta}{\partial A}\frac{dA^*}{dr^p} \\
&= \left(\frac{\partial \psi_\theta}{\partial r^p} - \frac{\psi_\theta}{1-r^p} - \frac{\psi_\theta}{A^*}\frac{dA^*}{dr^p}\right)X
\end{aligned}$$

Focus on the term in the parenthesis:

$$= \frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{1-r^p} - \frac{\frac{\psi_\theta}{A^*}\sum_{\theta=l,b} N_\theta W(\lambda_\theta)\left[\frac{\partial \psi_\theta}{\partial r^p}\mathbf{T}_\theta + \left(\psi_\theta + \mathbf{T}_\theta \frac{\partial \psi_\theta}{\partial T}\right)X\left(\frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{1-r^p}\right)\right]}{1+\sum_{\theta=l,b} N_\theta W(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}}.$$

When γ is small enough, then $\psi_b \to \psi_l$ and $\frac{\partial \psi_b}{\partial r^p} \to \frac{\partial \psi_l}{\partial r^p}$ so that the last line becomes

$$\begin{aligned}
&= \frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{1-r^p} - \frac{\frac{\psi_\theta}{A^*}NW(\lambda_\theta)\left[\frac{\partial \psi_\theta}{\partial r^p}\mathbf{T}_\theta + \left(\psi_\theta + \mathbf{T}_\theta \frac{\partial \psi_\theta}{\partial T}\right)X\left(\frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{1-r^p}\right)\right]}{1+NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}} \\
&= \frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{1-r^p} - \frac{\frac{\partial \psi_\theta}{\partial r^p} + \frac{\psi_\theta}{A^*}NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta \frac{\partial \psi_\theta}{\partial T}\right)X\left(\frac{\partial \psi_\theta}{\partial r^\eta} - \frac{\psi_\theta}{1-r^p}\right)}{1+NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}} \\
&= -\frac{\psi_\theta}{1-r^p} + \frac{NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}}{1+NW(\lambda_\theta)\left(\psi_\theta + \mathbf{T}_\theta\frac{\partial \psi_\theta}{\partial T}\right)X\frac{\psi_\theta}{A^*}}\frac{\psi_\theta}{1-r^p} \\
&< 0
\end{aligned}$$

Next, from (6) and utilizing Equation (20), from (5), the following must hold in the equilibrium

$$\begin{aligned}
\alpha_\theta &= \psi_\theta\left(T_\theta\right) \\
&\equiv \frac{(1-r^p)V_\theta\lambda_\theta - D_\theta(1-r^p)V_\theta\lambda_\theta}{2(1-r^p)V_\theta\lambda_\theta - D_\theta(1-r^p)V_\theta\lambda_\theta} \\
&= \frac{1-D_\theta}{2-D_\theta}
\end{aligned}$$

which implies that $\frac{dD_\theta}{dr^p}$ must have the opposite sign as $\frac{d\alpha_\theta}{dr^p}$. Moreover, if $\gamma \to 0$ then $D_\theta \to 1/N$, independent of r^η.■

A.4 Proof of Proposition 2

The proof follows directly from the main text.■

A.5 Proof of Proposition 3

The proof follows directly from the main text.■

A.6 Proof of Proposition 4

Given that $r^\eta = 1$ across both models, the price comparison follows directly from the logic in the text preceding the statement of the proposition. The increase in content created follows directly from Lemma 2, and the increase in total welfare follows from the fact that the monopolist, having all of the choices available to the duopolists along with their combined revenue along with the ability to coordinate prices means that the monopolist must be better off with these prices than it would be with the duopoly prices, so platform revenue is higher. The creators are better off because they face the same r^η and lower r^p, and consumers are better off because they have more content being created for them to watch.■

A.7 Proof of Lemma 3

Using Equation (5), it is clear from inspection that $\frac{\partial \mathbf{T}_b}{\partial W} > 0$, $\frac{\partial \mathbf{T}_l}{\partial W} = \frac{\partial \psi_l}{\partial W} = \frac{\partial \psi_b}{\partial W} = 0$, and $\frac{\partial \mathbf{T}_\theta}{\partial A} < 0$. From the definition of A^* in Equation (7), by implicit function theorem, $\frac{dA^*}{dW}$ has the same sign as

$$\psi_b \mathbf{T}_b + W \frac{\partial \psi_\theta \mathbf{T}_\theta}{\partial W} = \psi_\theta \mathbf{T}_\theta + W \left(\psi_b + \frac{\partial \psi_b}{\partial T} \mathbf{T}_b \right) \frac{\partial \mathbf{T}_b}{\partial W} > 0.$$

Next, $\frac{dA^*}{dW} > 0$ implies $\frac{dT_l}{dW} = \frac{\partial \mathbf{T}_l}{\partial A} \frac{dA^*}{dW} < 0$ and then Assumption 2 implies $\frac{d\alpha_l T_l}{dW} = \left(\psi_l + \frac{\partial \psi_l}{\partial T} \mathbf{T}_l \right) \frac{\partial \mathbf{T}_l}{\partial W} < 0$. Then, $D_l = \frac{\alpha_l T_l}{A^*}$ must be decreasing. Using $N_l D_l + N_b D_b = 1$ so that

$$N_l \frac{dD_l}{dW} + N_b \frac{dD_b}{dW} = 0$$

we conclude $\frac{dD_b}{dW} > 0$ in the equilibrium. Then, from Equation (20), we then have that $\frac{dT_b}{dW} > 0$, so that Assumption 2 implies $\frac{d\alpha_l T_l}{dW} > 0$.■

A.8 Proof of Proposition 5

Using $\frac{dD_l}{dD_b} = -\frac{N_b}{N_l}$ so that $\frac{d\alpha_l}{dD_b} = -\frac{N_b}{N_l} \frac{d\alpha_l}{dD_l}$ and $\frac{dT_l}{dD_b} = -\frac{N_b}{N_l} \frac{dT_l}{dD_l}$, the derivative of the profit function can be written as

$$\begin{aligned}
& \frac{1}{N_b} \frac{d\Pi^{hybrid}}{dD_b} \\
= \; & \left(\alpha_b \eta r^\eta + (1 - \alpha_b) V_b r^p \right) \lambda_b \left(T_b + \frac{dT_b}{dD_b} D_b \right) - \left(\alpha_l \eta r^\eta + (1 - \alpha_l) V_l r^p \right) \lambda_l \left(T_l + \frac{dT_l}{dD_l} D_l \right) \\
& + \left(\eta r^\eta - V_b r^p \right) \frac{d\alpha_b}{dD_b} T_b \lambda_b D_b - \left(\eta r^\eta - V_l r^p \right) \frac{d\alpha_l}{dD_l} T_l \lambda_l D_l
\end{aligned}$$

From $\alpha_\theta = \frac{1-D_\theta}{2-D_\theta}$ and $T_\theta = (C')^{-1}(D_\theta(1-r^p)\lambda_\theta V_\theta)$, we can calculate $\frac{d\alpha_\theta}{dD_\theta} = \frac{-1}{(2-D_\theta)^2} < 0$ and

$$\frac{dT_\theta}{dD_\theta} D_\theta = \frac{(1-r^p) V_\theta \lambda_\theta D_\theta}{C''(T_\theta)} = \frac{C'(T_\theta)}{C''(T_\theta)} = \frac{cT_\theta^2}{2cT_\theta} = \frac{T_\theta}{2} > 0,$$

where the last equality is due to $C(T_\theta) = \frac{c}{3} T_\theta^3$. Substituting these into the derivative, we get

$$\begin{aligned}
\frac{1}{N_b} \frac{d\Pi^{hybrid}}{dD_b} = \; & \left(\alpha_b \eta r^\eta + (1 - \alpha_b) V_b r^p \right) \left(\frac{3\lambda_b T_b}{2} \right) - \left(\alpha_l \eta r^\eta + (1 - \alpha_l) V_l r^p \right) \left(\frac{3\lambda_l T_l}{2} \right) \\
& + \left(V_b r^p - \eta r^\eta \right) \frac{T_b \lambda_b D_b}{(2 - D_b)^2} - \left(V_l r^p - \eta r^\eta \right) \frac{T_l \lambda_l D_l}{(2 - D_l)^2}.
\end{aligned} \tag{21}$$

By Lemma 3 and $\eta < r^p V_b < r^p V_l$, all four components in this expression is always increasing in D_b. Hence we conclude Π^{hybrid} is convex.

Given the convexity, the maximization problem boils down to comparing between the extreme points: $\Pi^{hybrid}_{D_b=1/N_b} > \Pi^{hybrid}_{D_l=1/N_l}$ (note $D_l = 1/N_l$ is equivalent to $D_b = 0$) if and only if

$$(\alpha_b \eta r^\eta + (1-\alpha_b) V_b r^p) T_b \lambda_b > (\alpha_l \eta r^\eta + (1-\alpha_l) V_l r^p) T_l \lambda_l.$$

Substuting for α_θ, T_θ, and D_θ, the inequality holds if and only if $Z_b > Z_l$, where

$$Z_\theta = \left(\left(\frac{N_\theta - 1}{2N_\theta - 1} \right) \lambda_\theta + \left(\frac{N_\theta}{2N_\theta - 1} \right) \left(\frac{r^p}{r^\eta \eta} \right) \lambda_\theta V_\theta \right) (C')^{-1} \left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta} \right),$$

as defined in (11), is written purely in terms of exogenous parameters. Obviously, $Z_b - Z_l$ is increasing in V_b and negative if $V_b \to 0$. If $Z_b - Z_l \leq 0$ for all $V_b < V_l$, then we let $\bar{V}^{hybrid} = V_l$. Otherwise, if $Z_b - Z_l > 0$ for $V_b \to V_l$, then by the intermediate value theorem, the required unique threshold $\bar{V}^{hybrid} \in [0, V_l]$ exists.

Next, $Z_b - Z_l$ is obviously decreasing in λ_l and V_l and N_b (given $r^\eta \eta < r^p V_b < r^p V_l$) so that the threshold $\bar{V}^{hybrid}$ increases with these parameters. Similarly, $Z_b - Z_l$ is increasing in λ_b and N_l and so the reverse is true for $\bar{V}^{hybrid}$.$\blacksquare$

A.9 Proof of Proposition 6

The calculation of the profit derivative is exactly the same as the proof of Proposition 5, except that we need to replace platform's margin from type-θ creators from $(\alpha_\theta \eta r^\eta + (1-\alpha_\theta) V_\theta r^p)$ to $\alpha_\theta \eta r^\eta$. Thus, the following equation is analogous to (21):

$$\begin{aligned} \frac{1}{N_b} \frac{d\Pi^{disc}}{dD_b} &= \alpha_b \eta r^\eta \left(\frac{3\lambda_b T_b}{2} \right) - \alpha_l \eta r^\eta \left(\frac{3\lambda_l T_l}{2} \right) \\ &\quad - \eta r^\eta \frac{T_b \lambda_b D_b}{(2-D_b)^2} + \eta r^\eta \frac{T_l \lambda_l D_l}{(2-D_l)^2}. \end{aligned}$$

Substituting $\alpha_\theta = \frac{1-D_\theta}{2-D_\theta}$ and rerranging, we have

$$\frac{1}{\eta r^\eta N_b} \frac{d\Pi^{disc}}{dD_b} = T_b Y(D_b) \lambda_b - T_l Y(D_l) \lambda_l,$$

where

$$Y(D) = \frac{3(1-D)}{2(2-D)} - \frac{D}{(2-D)^2}$$

is strictly positive for all $D < 1/2$. Using $C(T_\theta) = \frac{c}{3} T_\theta^3$, we have $(C')^{-1}(x) = \left(\frac{x}{c}\right)^{1/2}$, so

$$\begin{aligned} T_\theta Y(D_\theta) \lambda_\theta &= \left(\frac{D_\theta (1-r^p) V_\theta \lambda_\theta}{c} \right)^{1/2} \lambda_\theta \left(\frac{3(1-D_\theta)}{2(2-D_\theta)} - \frac{D_\theta}{(2-D_\theta)^2} \right) \\ &= \left(\frac{(1-r^p) V_\theta \lambda_\theta^3}{c} \right)^{1/2} \left(\frac{3(1-D_\theta)}{2(2-D_\theta)} - \frac{D_\theta}{(2-D_\theta)^2} \right) D_\theta^{1/2}. \end{aligned}$$

The multiplicative form means that whether $T_\theta Y(D_\theta)\lambda_\theta$ is increasing in D_θ or not is independent of θ. It is straightforward to verify that

$$\left(\frac{3(1-D)}{2(2-D)} - \frac{D}{(2-D)^2}\right) D^{1/2}$$

is increasing for all $D \leq 1/4$, so $T_\theta Y(D_\theta)\lambda_\theta$ is increasing for both $\theta = l, b$. It follows that $\frac{d\Pi^{disc}}{dD_b}$ is increasing in D_b if $N_b > 3$ and $N_l > 3$, which establishes convexity.

Given the convexity, the maximization problem boils down to comparing between the extreme points: $\Pi^{disc}_{D_b=1/N_b} > \Pi^{disc}_{D_l=1/N_l}$ if and only if

$$\alpha_b T_b \lambda_b > \alpha_l T_l \lambda_l.$$

Substuting for α_θ, T_θ, and D_θ, the inequality holds if and only if $\tilde{Z}_b > \tilde{Z}_l$, where

$$\tilde{Z}_\theta = \left(\frac{N_\theta - 1}{2N_\theta - 1}\right) \lambda_\theta (C')^{-1} \left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta}\right),$$

as defined in (12), is written purely in terms of exogenous parameters. Obviously, $\tilde{Z}_b - \tilde{Z}_l$ is increasing in V_b and negative if $V_b \to 0$. If $\tilde{Z}_b - \tilde{Z}_l \leq 0$ for all $V_b < V_l$, then we let $\bar{V}^{disc} = V_l$. Otherwise, if $\tilde{Z}_b - \tilde{Z}_l > 0$ for $V_b \to V_l$, then by the intermediate value theorem, the required unique threshold $\bar{V}^{disc} \in [0, V_l]$ exists.

Next, $\tilde{Z}_b - \tilde{Z}_l$ is obviously decreasing in λ_l and V_l so that the threshold $\bar{V}^{disc}$ increases with these parameters. Similarly, $\tilde{Z}_b - \tilde{Z}_l$ is increasing in λ_b, so $\bar{V}^{disc}$ is decreasing in λ_b. Finally, using the cubic cost function to rewrite

$$\tilde{Z}_\theta = \left(\frac{N_\theta - 1}{2N_\theta - 1}\right) \lambda_\theta \left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta c}\right)^{1/2}$$

where $\frac{d\tilde{Z}_\theta}{dN_\theta}$ has the same sign as

$$\frac{5N_\theta - 1 - 2N_\theta^2}{2N_\theta^{3/2}(2N_\theta - 1)^2} < 0$$

for all $N_\theta > 3$. So, $\tilde{Z}_b - \tilde{Z}_l$ is decreasing in N_b and increasing in N_l, and so the reverse is true for $\bar{V}^{disc}$. ■

A.10 Proof of Proposition 7

From the definition of Z_θ in (11), rewrite it as

$$\begin{aligned} Z_\theta &= \left(\lambda_\theta + \frac{N_\theta}{N_\theta - 1}\left(\frac{r^p}{r^\eta \eta}\right)\lambda_\theta V_\theta\right)\left(\frac{N_\theta - 1}{2N_\theta - 1}\right)(C')^{-1}\left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta}\right) \\ &= \left(1 + \frac{N_\theta}{N_\theta - 1}\left(\frac{r^p}{r^\eta \eta}\right) V_\theta\right)\tilde{Z}_\theta. \end{aligned}$$

Then,

$$Z_b - Z_l = \left(1 + \frac{N_b}{N_b - 1}\left(\frac{r^p}{r^\eta \eta}\right) V_b\right)\tilde{Z}_b - \left(1 + \frac{N_l}{N_l - 1}\left(\frac{r^p}{r^\eta \eta}\right) V_l\right)\tilde{Z}_l \tag{22}$$

Evaluating at $V_b = \bar{V}^{disc}$, by definition we have $\tilde{Z}_b = \tilde{Z}_l$, so that $Z_b - Z_l < 0$ if and only if condition (13) holds. Given that $Z_b - Z_l$ is monotone increasing in V_b, if condition (13) holds then $Z_b < Z_l$ for all $V_b \le \bar{V}^{disc}$, meaning that $\bar{V}^{hybrid} > \bar{V}^{disc}$, and vice-versa. Note that we can equivalently evaluate (22) at $V_b = \bar{V}^{hybrid}$ and arrive at an equivalent condition of

$$\frac{N_l V_l}{N_l - 1} > \frac{N_b \bar{V}^{hybrid}}{N_b - 1}.$$

■

A.11 Proof of Proposition 8

Rewrite

$$\Omega = \frac{1}{r^\eta}\Pi^{disc} + N_b(D_b\lambda_b V_b T_b - C(T_b)) + N_l(D_l\lambda_l V_l T_l - C(T_l))$$

Taking the derivative of welfare with regard to D_b and using $\frac{dD_l}{dD_b} = -\frac{N_b}{N_l}$ and $\frac{dT_l}{dD_b} = -\frac{N_b}{N_l}\frac{dT_l}{dD_l}$, we get

$$\begin{aligned}\frac{1}{N_b}\frac{d\Omega}{dD_b} \quad &= \quad \frac{1}{N_b r^\eta}\frac{d\Pi^{disc}}{dD_b} + \left(\lambda_b V_b T_b + (\lambda_b V_b D_b - C'(T_b))\frac{dT_b}{dD_b}\right) \\ &\quad - \left(\lambda_l V_l T_l + (\lambda_l V_l D_l - C'(T_l))\frac{dT_l}{dD_l}\right)\end{aligned}$$

Using the equilibrium relationship $(1 - r^p)V_\theta \lambda_\theta D_\theta = C'(T_\theta)$, we get

$$\frac{1}{N_b}\frac{d\Omega}{dD_b} = \frac{1}{N_b r^\eta}\frac{d\Pi^{disc}}{dD_b} + \lambda_b V_b\left(T_b + r^p D_b\frac{dT_b}{dD_b}\right) - \lambda_l V_l\left(T_l + r^p D_l\frac{dT_l}{dD_l}\right)$$

We already know that $N_b > 3$ and $N_l > 3$ are sufficient for Π^{disc} to be convex, so $\frac{d\Pi^{disc}}{dD_b}$ is increasing. Therefore, a sufficient condition for welfare convexity is

$$T_\theta + r^p D_\theta\frac{dT_\theta}{dD_\theta} = T_\theta + r^p\frac{C'(T_\theta)}{C''(T_\theta)}$$

being increasing in T_θ. So, $\frac{d}{dx}\frac{C'(x)}{C''(x)} > \frac{-1}{r^p}$ is sufficient for convexity. In the case of cubic costs $\frac{C'(x)}{C''(x)} = \frac{x}{2} > 0$, so welfare is convex.

Analagously to the previous proofs, define $\hat{Z}_\theta$ as

$$\hat{Z}_\theta = \lambda_\theta\left(V_\theta + \left(\frac{N_\theta - 1}{2N_\theta - 1}\right)\eta\right)C'^{-1}\left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta}\right) - N_\theta C\left(C'^{-1}\left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta}\right)\right)$$

Taking derivatives, it is easy to verify that $\hat{Z}_\theta$ is increasing in λ_θ and V_θ.

Obviously, $\hat{Z}_b - \hat{Z}_l$ is negative if $V_b \to 0$. If $\hat{Z}_b - \hat{Z}_l \le 0$ for all $V_b < V_l$, then we let $\bar{V}^{welfare} = V_l$. Otherwise, if $\hat{Z}_b - \hat{Z}_l > 0$ for $V_b \to V_l$, then by the intermediate value theorem, the required unique threshold $\bar{V}^{welfare} \in [0, V_l]$ exists. Finally, given $\hat{Z}_\theta$ is increasing in λ_θ and V_θ, the threshold $\bar{V}^{welfare}$ is increasing in V_l and λ_l and decreasing in λ_b.■

A.12 Proof of Proposition 9

To simplify reading the derivation, recall that $T_\theta = C'^{-1}\left(\frac{(1-r^p)\lambda_\theta V_\theta}{N_\theta}\right)$, so we can rewrite

$$\tilde{Z}_\theta = \left(\frac{N_\theta - 1}{2N_\theta - 1}\right)\lambda_\theta T_\theta$$

and

$$\begin{aligned}\hat{Z}_\theta =& \frac{V_\theta + \frac{N_\theta-1}{2N_\theta-1}\eta}{\left(\frac{N_\theta-1}{2N_\theta-1}\right)}\tilde{Z}_\theta - N_\theta C\left(T_\theta\right)\\ =& \left(\frac{2N_\theta - 1}{N_\theta - 1}V_\theta + \eta\right)\tilde{Z}_\theta - N_\theta C\left(T_\theta\right).\end{aligned}$$

So

$$\begin{aligned}\hat{Z}_b - \hat{Z}_l = & \left(\frac{2N_b - 1}{N_b - 1}V_b + \eta\right)\tilde{Z}_b - \left(\frac{2N_l - 1}{N_l - 1}V_l + \eta\right)\tilde{Z}_l\\ & + N_l C\left(T_l\right) - N_b C\left(T_b\right)\end{aligned}$$

Evaluating at $V_b = \bar{V}^{welfare}$

$$\begin{aligned}&\left(\frac{2N_b - 1}{N_b - 1}\bar{V}^{welfare} + \eta\right)\tilde{Z}_b - \left(\frac{2N_l - 1}{N_l - 1}V_l + \eta\right)\tilde{Z}_l\\ =& N_b C\left(T_b\right) - N_l C\left(T_l\right)\end{aligned}$$

Which then implies

$$\tilde{Z}_b = \frac{\left(\frac{2N_l-1}{N_l-1}V_l + \eta\right)\tilde{Z}_l + N_b C\left(T_b\right) - N_l C\left(T_l\right)}{\left(\frac{2N_b-1}{N_b-1}\bar{V}^{welfare} + \eta\right)}$$

Dividing through by $\tilde{Z}_l$

$$\frac{\tilde{Z}_b}{\tilde{Z}_l} = \frac{\left(\frac{2N_l-1}{N_l-1}V_l + \eta\right)\tilde{Z}_l + N_b C\left(T_b\right) - N_l C\left(T_l\right)}{\tilde{Z}_l\left(\frac{2N_b-1}{N_b-1}\bar{V}^{welfare} + \eta\right)}$$

If the left hand side is greater than 1, then this implies $\tilde{Z}_b > \tilde{Z}_l$, so $\bar{V}^{welfare} \geq \bar{V}^{disc}$ if and only if

$$\frac{\left(\frac{2N_l-1}{N_l-1}V_l + \eta\right)\tilde{Z}_l + N_b C\left(T_b\right) - N_l C\left(T_l\right)}{\tilde{Z}_l\left(\frac{2N_b-1}{N_b-1}\bar{V}^{welfare} + \eta\right)} \geq 1$$

Which can be simplified to condition (14).

Running through similar steps for Z_θ, whereby

$$\hat{Z}_\theta = \left(\frac{\frac{2N_\theta-1}{N_\theta-1}V_\theta + \eta}{1 + \frac{N_\theta}{N_\theta-1}\left(\frac{r^p}{r^\eta\eta}\right)V_\theta}\right)Z_\theta - N_\theta C\left(T_\theta\right),$$

we have $\bar{V}^{welfare} \geq \bar{V}^{hybrid}$ if and only if condition (15) holds. ■

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