Online Appendix

Content-hosting platforms: discovery, membership, or both?

Ben Casner and Tat-How Teh

A Extra details: monopoly benchmark (Section 4)

Consider the monopoly model in Section 3. An alternative formulation for the utility realization in (1) is to allow for idiosyncratic match value realizations whenever a match in taste occurs à la Eliaz and Spiegler (2011) and Yang (2013). Specifically, rewrite (1) as

$$u_{ij} = \left\{ \begin{array}{cc} y_{ij} + u(\lambda_i) & \text{with probability } \lambda_i \\ 0 & \text{with probability } 1-\lambda_i \end{array} \right\},$$

where $y_{ij} \in [0, y_{\max}]$ is a pair-ij-specific idiosyncratic factor that is i.i.d. according to a log-concave CDF function F. Notice that if $y_{\max} \to 0$ so that CDF F is degenerate, then this formulation nests (1) as a special case.

The realization y_{ij} is privately observed only by the consumer and only after inspection. In particular, if one goes with the microfoundation presented in Section A, then it means that the platform never observes the realizations of y_{ij}, thus preventing the correlation structure in the consumer search sequence. Therefore, the consumer search environment remains stationarity. Moreover, platforms are unable to condition its recommendation based on the realization of y_{ij}, and so the recommendation (2) remains applicable and consistent with the Pandora's rule (Weitzman 1979).

Then, we obtain the following equivalence of Lemma 1:

Lemma A.1. *(Hybrid mode subgame). For each given (r, τ), the equilibrium of the consumer-creator subgame is:*

1. *Each creator i joins both portals of the platform and sets equilibrium design*

$$\lambda^* = \arg\max_{\lambda_i \in [0,1]} \left\{ D\left(\frac{\tilde{u}(\lambda_i)}{1-r}; \frac{\tilde{u}(\lambda^*)}{1-r}\right) \times \left(1 - F(\bar{y} + u(\lambda^*) - u(\lambda_i))\right) \times \left(a + (1-\tau)v(\lambda_i)\right)\lambda_i \right\} \quad \text{(A.1)}$$

   where $\tilde{u}(\lambda_i)$ is the search reservation value used by platform for recommendations:

$$\tilde{u}(\lambda_i) \equiv u(\lambda_i) - \frac{s}{\lambda_i};$$

   whereas $\bar{y}$ is the on-equilibrium-path search reservation value used by consumers when deciding whether to stop:

$$\int_{\max\{\bar{y},0\}}^{x_{\max}} (y - \bar{y}) dF(y) = \frac{s}{\lambda^*}.$$

2. *Each consumer believes that all creators adopt strategy λ^* in (6), and initiates search if and only if $x \le \bar{y} + \tilde{u}(\lambda^*)$, and does so through the discovery portal (follows the recommendation in every step of search). The mass of consumers who search is $G(\bar{y} + \tilde{u}(\lambda^*))$.*

3. *Each consumer j stops searching and becomes a viewer of the creator i if and only if (i) there is a match in taste (so $u_{ij} \neq 0$), and (ii) the realized y_{ij} satisfies*

$$y_{ij} + u(\lambda_i) \ge \bar{y} + u(\lambda^*). \quad \text{(A.2)}$$

Proof. Conditioned on consumer search stopping rule, as represented by cutoff $\bar{x}$, the equilibrium construction follows directly from Lemma 1. Meanwhile, the proof of consumer search stopping rule is a direct extension of Wolinsky (1986), as noted in Proposition 1 of Eliaz and Spiegler. ■

Observe that if $y_{\max} \to 0$, then $\bar{y} = -\frac{s}{\lambda^*}$, and so $1 - F(\bar{x} + u(\lambda^*) - u(\lambda_i)) = 1$ in (A.1). In this case, (A.2) always holds and so Lemma A.1 reduces to Lemma 1. Hence, an interpretation of the model in Section 3 is it represents a limiting case where the idiosyncratic search component of consumer utility, i.e., y_{ij}, is sufficiently homogeneous so that consumers stop searching at the first match in taste.

More generally, the additional component $1 - F(\bar{y} + u(\lambda^*) - u(\lambda_i)) < 1$ in (A.1) induces equilibrium design that is less broad compared to the one in the main text (6). Intuitively, the additional component $1 - F(\bar{y} + u(\lambda^*) - u(\lambda_i))$ means that creators have an incentive to lower broadness in order to increase the likelihood of retaining positively-matched consumers.

Nonetheless, the equilibrium outcome in Lemma A.1 is conceptually the same as Lemma 1. In particular, the comparative statics in Proposition 1 on the effects of platform decisions r and τ remain valid. In light of this, we stick with the more tractable formulation in Section 3 in order to operationalize the analysis of platform decisions.

Lemma A.2. *If $\frac{a'(\lambda_i)}{a(\lambda_i)} \geq \frac{v'(\lambda_i)}{v(\lambda_i)}$ holds for all λ_i, then λ^* is increasing in τ.*

Proof. Following the same steps as the proof of Lemma 1, denote λ^* as the solution to FOC

$$0 = \frac{1}{\lambda^*} + \frac{1}{1-r}\frac{D'}{D}\frac{\partial \tilde{u}_i}{\partial \lambda_i} + \frac{a'(\lambda^*) + (1-\tau)v'(\lambda^*)}{a(\lambda^*) + (1-\tau)v(\lambda^*)}.$$

By total differentiation, the signs of $\partial\lambda^*/\partial\tau$ is the same as

$$\frac{\partial}{\partial \tau}\left(\frac{a'(\lambda^*) + (1-\tau)v'(\lambda^*)}{a(\lambda^*) + (1-\tau)v(\lambda^*)}\right) = \frac{a'(\lambda^*)v(\lambda^*) - a(\lambda^*)v'(\lambda^*)}{\left(a + (1-\tau)v(\lambda^*)\right)^2} \geq 0$$

if the stated condition holds. ■

B Extra details: competing platforms (Section 5)

B.1 Sequential business model choice

Consider the complete multihoming competition model in Section 5 but with sequential business mode decisions. Platform 1 (first mover) decides on a business model first before Platform 2 (follower) and then the rest of the game proceeds as before. Using Proposition 4 and taking into account the backward induction logic, the following result is immediate:

Proposition B.1. *Suppose platforms make sequential choices of business model with P_1 being the first mover, there exists a threshold $A_{\text{seq}} \leq A_{MH}$ such that:*[33]

- *If $A \geq A_{MH}$, then Platform 1 optimally chooses H and induces $P_2 = D$.*
- *If $A \in (A_{\text{seq}}, A_{MH})$, then Platform 1 optimally chooses D and induces $P_2 = M$.*
- *If $A \leq A_{\text{seq}}$, then Platform 1 optimally chooses M and induces $P_2 = D$.*

[33] If $A_{seq} = A_{MH}$, then (A_{seq}, A_{MH}) is an empty set. A simple sufficient condition for $A_{seq} < A_{MH}$ is $v(0)$ being sufficiently small relative to A.

Proof. (**Proposition B.1**). When $A \geq A_{MH}$, the backward induction logic and the best response of the follower (Proposition 4) imply Platform 1 essentially compares between $\Pi_1^*(M, D)$, $\Pi_1^*(H, D)$, and $\Pi_1^*(D, H)$. Then, Proposition 3 implies $\Pi_1^*(H, D) > \Pi_1^*(D, H)$, whereas the definition of A_{MH} implies $\Pi_1^*(H, D) \geq \Pi_1^*(M, D)$.

When $A < A_{MH}$, we first rule out $P_1 = H$ being optimal. To see this, if $A \in [A'_{MH}, A_{MH})$, then $P_2 = D$ in response, and the definition of A_{MH} implies $\Pi_1^*(H, D) < \Pi_1^*(M, D)$; if $A < A'_{MH}$ in response, then $P_2 = M$, and Proposition 4 implies $\Pi_1^*(H, M) < \Pi_1^*(D, M)$ in this case. Therefore, Platform 1 essentially compares between $\Pi_1^*(M, D)$ and $\Pi_1^*(D, M)$ (recall $A < A_{MH}$ means $P_2 = M$ in response to $P_1 = D$). Recall

$$\begin{aligned}
\Pi_1^*(D, M) &= G(\tilde{u}(\lambda^*(\bar{r}, \tau_{DM}))A - C \\
\Pi_1^*(M, D) &= G(\tilde{u}(\lambda^*(\bar{r}, \tau_{MD}))\tau_{MD} v(\lambda^*)
\end{aligned}$$

Notice $\tau_{DM} = \tau_{MD}$ is chosen by the platform with $P_l = M$ and so it is independent of A, implying that $\Pi_1^*(D, M) - \Pi_1^*(M, D)$ is monotone increasing in A.[34] If $\Pi_1^*(D, M) < \Pi_1^*(M, D)$ for all $A < A_{MH}$, then we define $A_{seq} = A_{MH}$. Otherwise, define A_{seq} as the solution to $\Pi_1^*(D, M) = \Pi_1^*(M, D)$, which exists and is unique by the intermediate value theorem. ■

Proposition B.1 says that the first mover monopolizes the membership portal market by operating an membership portal preemptively if ad profitability A is either very large or very small. For some parameter configurations, there exists an intermediate region of A such that $\Pi_1^*(M, D) < \Pi_1^*(D, M)$, in which case the first mover does not monopolize the membership portal market as it earns a higher profit by being the sole discovery portal.

B.2 Differentiated membership portals

Consider the complete multihoming competition model in Section 5 but change it such that creators now view membership portals as being horizontally differentiated. All timing is identical to the model in Section 5. Note this extension involves potential asymmetric design on the equilibrium path, similar to the extension in Section E. For tractability, we focus on the logit recommendation form in 5 and focus on the case where G is sufficiently inelastic throughout.

We adopt the content differentiation scheme of Wang and Wright (2020). Whenever two membership portals coexist, half of creators join each membership portal as a "default". If they consider switching to the alternative portal they face a switching cost σz where $z \in [\underline{z}, \bar{z}]$ is distributed according to CDF Q whereas $\sigma \in (0, \infty]$ represents the degree of differentiation between platforms. Clearly, $\sigma \to 0$ recovers the model in Section 5.

In what follows, we focus on $\sigma \to \infty$, i.e., membership portals are local monopolies. Likewise, if there is only one membership portal, all creators join the sole membership portal as the "default" and face no switching cost (as there is no where else to switch to). As such, the creator's maximization problem is essentially unchanged after the participation step.

Consider any subgame where two platforms operate two coexisting membership portals. Let the set of creators who joins the membership portal of platform 1 be S_1 (all these creators choose $\lambda_1^* = \lambda^*(r, \tau_1)$) and the set of sellers who join the membership portal of platform 2 be S_2 (all these creators optimally choose $\lambda_2^* = \lambda^*(r, \tau_2)$), where $r = \max\{r_1, r_2\}$, similar to Lemma 2. By the local monopolies assumption, creators split themselves across membership portals evenly so $|S_1| = |S_2| = 1/2$. Then, the *total probability*

[34] Recall the alphabetical subscript in τ denotes the mode choices P_1 and P_2 in successive order.

of creators in set S_l ($l = 1, 2$) being recommended is

$$D_l = \frac{\frac{1}{2}\exp\left(\frac{\tilde{u}(\lambda_l^*)}{1-r}\right)}{\frac{1}{2}\sum_{l=1,2}\exp\left(\frac{\tilde{u}(\lambda_l^*)}{1-r}\right)}.$$

Using the terminologies developed in Section E, let $\tilde{F}$ be the (recommendation-weighted) *effective distribution of content design*, which is a binary distribution. The corresponding probability mass function is

$$\begin{aligned} \tilde{f}(z) &= D_1 \text{ if } z = \lambda_1^* \\ \tilde{f}(z) &= D_2 \text{ if } z = \lambda_2^* \end{aligned} \tag{B.1}$$

which is a function of (r, τ_1, τ_2) via $\lambda_1^* = \lambda^*(r, \tau_1)$ and $\lambda_2^* = \lambda^*(r, \tau_2)$. Following Lemma E.2, we know for given r on the equilibrium path, each consumer believes that the distribution of design is described by CDF $\tilde{F}$, and initiates search if and only if

$$\begin{aligned} x &\leq \tilde{u}_{12}(r, \lambda_1^*, \lambda_2^*) \\ &\equiv \tilde{u}(\lambda_1^*)D_1 + \tilde{u}(\lambda_2^*)D_2. \end{aligned}$$

and does so through the discovery portal (follows the recommendation in every step of search). Upon observing a positive match value with a creator, the consumer stops searching and becomes a viewer of the creator. Before proceeding, it is useful to note that

$$\frac{\partial D_l}{\partial \tau_l} > 0$$

due to assumption $s > \bar{s}_{max}$.

Clearly, the extension does not affect the maximization problem faced by a platform l if it chooses $P_l = D$. As for $P_l = M$ and H. The platform's tradeoff is similar to the monopoly problem, except that the addition of the rival membership portal means that platform l must now additionally take into account the effect of changing λ_l^* on D_l (the probability of "its creators" being recommended instead of "the rival's creators"). Formally,[35]

$$\Pi_M(\tau_l) = G(\tilde{u}_{12})D_l\tau_l v(\lambda_l^*) \quad \text{if } P_{-l} \in \{M, H\}$$

and $\Pi_H(r_l, \tau_l)$ can similarly be rewritten:

$$\begin{aligned} \Pi_H(r_l, \tau_l) &= G(\tilde{u}_{12})\left(D_l\tau_l v(\lambda_l^*) + A\right) \text{ if } P_{-l} = M \\ &= G(\tilde{u}_{12})\left(D_l\tau_l v(\lambda_l^*) + \frac{A}{2}\right) \text{ if } P_{-l} = H \end{aligned}$$

Observe that in both expression of Π_M and Π_H, $\tau_l = 0$ is never optimal because G being sufficiently inelastic means

$$\frac{\partial \Pi_M}{\partial \tau_l}|_{\tau_l=0} = \frac{\partial \Pi_H}{\partial \tau_l}|_{\tau_l=0} = G(\tilde{u}_{12})D_l v(\lambda_l^*) > 0$$

Therefore, in any equilibrium with two coexisting membership portals, we must have $\tau_1 > 0$ and $\tau_2 > 0$.

Meanwhile, similar to the proof of Proposition 4 we know Bertrand competition between two homogeneous discovery portal components (whenever they coexist) to attract consumer search (and to earn the platform advertising revenue $A > 0$) implies $r_1 = r_2 = \bar{r}$. There is no incentive to deviate by lowering r_l because that doing so does not affect creators' content design and hence does not affect consumers' search decisions.

[35] Implicitly, we assume throughout that all the required second order conditions hold. In earlier versions, we also characterized sufficient conditions for $\Pi_M(\tau_l)$ and $\Pi_H(r_l, \tau_l)$ to be concave with respect to τ_l. The details are available from the authors upon request.

We now consider the choice of business models as a best response to another platform's choice of business model. Similar to the analysis in the proofs in Section 5, we first prove that $P_l = D$ is never a best response for any $P_{-l} \in \{M, D, H\}$. generality.

Lemma B.1. *If G is sufficiently inelastic , then $P_l = D$ is never a best response.*

Proof. We let platform indices $l = 2$ and $-l = 1$ without loss of generality. We want to show $P_2 = D$ is always dominated by $P_2 = H$. Suppose $P_1 = M$, and let $(\tau_{1,MH}, \tau_{2,MH})$ and r_{MH} denote platforms' equilibrium commissions and sensitivity when $(P_1, P_2) = (M, H)$. Using the fact that G is sufficiently inelastic, that $D_2 > 0$, and that $\tau_2 > 0$ in any configurations of business models, we have

$$\begin{aligned}\Pi^*_{2,MH} &= G \times (A + D_2 \tau_{2,MH} v(\lambda^*(r_{MH}, \tau_{2,MH}))) - C \\ &> G \times A - C = \Pi^*_{2,MD}.\end{aligned}$$

The same argument applies to $P_1 = D$ and $P_1 = H$. ■

To see why $P_l = D$ is never a best response in this setup, consider introducing $P_l \in \{H, M\}$ when $P_{-l} = M$ (same intuition for $P_{-l} = H$). Doing so has two effects on commissions. First, the coexistence of two membership portals (each with loyal creators) creates competition for recommendations on the part of the platforms because platforms earn commission revenue only if their loyal creators are recommended. This incentivizes platforms to increase τ to induce content design that appeals to the recommendation algorithm. Second, the coexistence of two membership portals pushes commissions down because consumers' participation decision is based on the *average* reservation value $\tilde{u}_{12}$ in the market, meaning that any unilateral changes in τ_l by platform l have a smaller downside (relative to the monopoly case) because creators that are loyal to platform $-l$ respond only to τ_{-l}. If G is sufficiently inelastic, the first effect dominates and the equilibrium commissions τ become higher compared to when there's only one membership portal. This shuts down the "negative spillover" discussed in 5 (i.e., when competition between membership portals reduces broadness).

Given that $P_l = D$ and/or $P_{-l} = D$ can never be part of the equilibrium, in the best-response analysis below we focus on the remain two modes: $P_l, P_{-l} \in \{M, H\}$

Proposition B.2. *If G is sufficiently inelastic, there are thresholds $A_{hori} \geq A'_{hori} \geq 0$ such that:*

- *If $P_{-l} = M$, Platform l optimally chooses H if $A \geq A_{hori}$ and chooses M if $A \leq A_{hori}$;*
- *If $P_{-l} = H$, Platform l optimally chooses H if $A \geq A'_{hori}$ and chooses M if $A \leq A'_{hori}$;*

Proof. We let platform indices $l = 2$ and $-l = 1$ without loss of generality. Consider $P_1 = M$. We want to show $\Pi_{2,MH} - \Pi_{2,MM}$ is monotone increasing in A where

$$\begin{aligned}\Pi_{2,MH} &= G\left(\tilde{u}_{12}(r_{MH}, \lambda^*(r_{MH}, \tau_{1,MH}), \lambda^*(r_{MH}, \tau_{2,MH}))\right) \times (A + D_2 \tau_{2,MH} v(\lambda^*(r_{MH}, \tau_{2,MH}))) - C \\ \Pi_{2,MM} &= G\left(\tilde{u}(\lambda^*(-\infty, \tau_{MM})\right) \times \frac{1}{2} \tau_{MM} v(\lambda^*(-\infty, \tau_{MM})).\end{aligned}$$

Obviously, $\Pi_{2,MM}$ is independent of A. Meanwhile, if G is sufficiently inelastic, then the duplet $(\tau_{1,MH}, \tau_{2,MH})$ is pinned down by the following FOCs:

$$\begin{aligned}1 + \tau_{1,MH}\left(\frac{dD_1}{d\tau_1}\frac{1}{D_1} + \frac{v'(\lambda_1^*(r_{MH}, \tau_{1,MH}))}{v(\lambda_1^*(r_{MH}, \tau_{1,MH}))}\frac{\partial \lambda_1^*}{\partial \tau_1}\right) &= 0 \\ 1 + \tau_{2,MH}\left(\frac{dD_2}{d\tau_2}\frac{1}{D_2} + \frac{v'(\lambda_2^*(r_{MH}, \tau_{2,MH}))}{v(\lambda_2^*(r_{MH}, \tau_{2,MH}))}\frac{\partial \lambda_2^*}{\partial \tau_2}\right) &= 0\end{aligned}$$

which are independent of A. Consequently, $\Pi_{2,MH}$ is monotone increasing in A by the envelope theorem.

Next, when $P_1 = H$. We want to show $\Pi_{2,HH} - \Pi_{2,HM}$ is monotone increasing in A, where

$$\begin{aligned}\Pi_{2,HH} &= G\left(\tilde{u}(\lambda^*(\bar{r},\tau_{HH})\right) \times \left(\frac{A}{2} + \frac{1}{2}\tau_{HH}v(\lambda^*(\bar{r},\tau_{HH}))\right) - C \\ \Pi_{2,HM} &= \left(\tilde{u}_{12}(r_{MH},\lambda^*(r_{MH},\tau_{1,MH}),\lambda^*(r_{MH},\tau_{2,MH}))\right) \times D_2\tau_{2,MH}v(\lambda^*(r_{MH},\tau_{2,MH})).\end{aligned}$$

Again, if G is sufficiently inelastic, the FOCs associated with $(\tau_{1,MH}, \tau_{2,MH})$ and τ_{HH} are independent of A. Hence, $\Pi_{2,HH}$ is increasing in A while $\Pi_{2,HM}$ is independent of A, as required.

Indeed, if G is fixed, we can solve for the cutoffs A_{hori} and A'_{hori} explicitly, and show that A'_{hori} is not always higher than A_{hori}. ∎

The best responses from Proposition B.2 lead immediately to the following result that is reminiscent of Proposition 7.

Corollary B.1. *Suppose G is sufficiently inelastic. In the equilibrium of the overall game:*[36]

- *If $A \geq A'_{hori}$, then both platforms operate in hybrid mode.*
- *If $A \in (A_{hori}, A'_{hori})$, one platform operates in pure membership mode and the other platform operates in hybrid mode.*
- *If $A \leq A_{hori}$, then both platforms operate in pure membership mode.*

C Extension: surplus and welfare implications

We include the proof of Corollary 3 here. Interested readers can find a more general exploration of welfare in this model in Section C.1.

Proof. (**Corollary 3**) Recall $s > \bar{s}_{\max}$ so that $\tilde{u}(\lambda_i)$ is increasing, and from Proposition 1 $\lambda^*_M \leq \lambda^*_H \leq \lambda^*_D$. The result follows. Creator surplus in the various modes is given by the following equations

$$\begin{aligned}PS_D &= G(\tilde{u}(\lambda^*_D))a \\ PS_H &= G(\tilde{u}(\lambda^*_H))(a + (1-\tau^*_H)v(\lambda^*_H)) \\ PS_M &= G(\tilde{u}_0(\lambda^*_M))(a + (1-\tau^*_M)v(\lambda^*_M)).\end{aligned}$$

Trivially $a + (1-\tau^*_H)v(\lambda^*(r^*_H,\tau^*_H)) > a$, so creators' *per-viewer* revenue is greater with a hybrid platform than under pure discovery. However, $\lambda^*_H \leq \lambda^*_D$ implies $G(\tilde{u}(\lambda^*_D)) \geq G(\tilde{u}(\lambda^*_H))$ so the mass of viewers is lower with a hybrid platform. However, from (8) as $\max_{\lambda_i}|\frac{v'(\lambda_i)}{v(\lambda_i)}| \to 0$, it must be the case that $\lambda^*_D \to \lambda^*_H$, so if $\max_{\lambda_i}|\frac{v'(\lambda_i)}{v(\lambda_i)}|$ is not too large then then the reduction in viewership must be outweighed by the increase in revenue because $\tilde{u}(\lambda^*)$ cannot have changed significantly. In that case $PS_H > PS_D$. To show that $PS_D \geq PS_M$ for large n_0, we note that as $\tilde{u}_0(.)$ is decreasing in n_0 whereas $\tilde{u}(.)$ does not depend on n_0, there must be n_0 sufficiently large that $a \cdot [G(\tilde{u}(\lambda^*_D)) - G(\tilde{u}_0(\lambda^*_M))] > v(\lambda^*_D)G(\tilde{u}(\lambda^*_D))$. If this inequality holds then $PS_D \geq PS_M$, regardless of the value of τ^*_M. More generally, from the FOC definition of τ^*_M:

$$\frac{G\left(\tilde{u}_0\left(\lambda^*\right)\right)}{g\left(\tilde{u}_0\left(\lambda^*\right)\right)}\left(v(\lambda^*) + v'(\lambda^*)\tau\frac{\partial\lambda^*}{\partial\tau}\right) + (\tau v(\lambda^*))\frac{\partial\tilde{u}_0}{\partial\lambda_i}\frac{\partial\lambda^*}{\partial\tau} = 0.$$

$\frac{\partial^2\tilde{u}_0}{\partial\lambda_i\partial n_0} > 0$, so τ^*_M is increasing in n_0, meaning that the intensive revenue gain from having a membership portal is diminished even further as n_0 increases. ∎

[36] Note that it is possible that $A_{hori} \geq A'_{hori}$, in which case we have coexistence of two types of equilibria: $(P_l, P_{-l}) = (H,H)$ and (M,M).

The MATLAB code used to generate Figures 2 and 3 are available from the authors upon request.

C.1 Generalized surplus and welfare implications

In the main body we presented a sufficiency result to generate orderings of consumer and producer welfare under the three business modes. The results contained in this section represent a more fulsome exploration of welfare in this model dropping the assumptions we use in the main text.

Continue from 6, recall

$$\begin{aligned} CS &\equiv \tilde{u}(\lambda^*)G(\tilde{u}(\lambda^*)) + \int_{\tilde{u}(\lambda^*)}^{\infty} xg(x)dx \\ PS &= G(\tilde{u}(\lambda^*))(a + (1-\tau)v(\lambda^*)). \end{aligned}$$

Let $\arg\max_\lambda \tilde{u}(\lambda) \equiv \lambda_{CS}$. The next two corollaries are generalizations of Corollary 3

Corollary C.1. *In the benchmark monopoly model:*

1. *A shift from pure discovery mode to hybrid mode weakly decreases (increases) consumer surplus if the CS-maximizing content design is sufficiently broad (niche):* $CS_H \leq CS_D$ *if* $\lambda_{CS} \geq \lambda^*(\bar{r}, 1)$*, otherwise* $CS_H \geq CS_D$*;*
2. *If A is sufficiently large then a shift from pure membership mode to hybrid mode weakly increases consumer surplus:* $CS_H \geq CS_M$*.*

Proof. The proofs of Corollary C.1 and C.2 below make extensive use of the following result: λ_{CS} is determined by the FOC

$$u'(\lambda) + \frac{s}{\lambda^2} = 0 \tag{C.1}$$

$u(\cdot)$ decreasing and concave implies that the second order condition is satisfied and that λ_{CS} is increasing in s.

The pure discovery platform's revenue is maximized when consumer surplus is maximized, therefore we must have $r_D = \bar{r}$. From Proposition 1 $\tilde{u}(\lambda_D)$ is increasing in r, whereas $\lambda_H \leq \lambda_D$. Concavity of $u(\cdot)$ implies that $\tilde{u}(\cdot)$ is concave and therefore has a unique maximum. If if $\lambda_{CS} \geq \lambda^*(\bar{r}, 1)$ then $\lambda_H \leq \lambda_D$ implies that $\tilde{u}(\lambda_H) \leq \tilde{u}(\lambda_D)$ and therefore $CS_H \leq CS_D$. If $\lambda_{CS} \leq \lambda^*(\bar{r}, 1)$ then an almost identical chain of logic implies $CS_H \geq CS_D$.

For part 2 of the corollary, consumer surplus would increase even if there were no change in creator behavior because of the decrease in effective search cost resulting from the recommendations. However, if we compare the designs between the two modes

$$\lambda_M = \arg\max_{\lambda_i \in [0,1]} \{\lambda_i \times (a + (1-\tau)v(\lambda_i))\}$$

$$\lambda_H = \arg\max_{\lambda_i \in [0,1]} \left\{ D\left(\frac{\tilde{u}(\lambda_i)}{1-r}; \frac{\tilde{u}(\lambda^*)}{1-r}\right) \lambda_i \left(a + (1-\tau)v(\lambda_i)\right) \right\}.$$

The latter is increasing in $\tilde{u}(\lambda_i)$ whereas the former is unaffected by consumer welfare. More formally, note that from the definition of λ_H and λ_M, if a creator designs content such that the FOC for λ_M is satisfied, then the derivative of their profit in the hybrid regime will be proportional to

$$\frac{1}{1-r} \frac{D'}{D} \frac{\partial \tilde{u}}{\partial \lambda}$$

and so from concavity of creator profit and $\tilde{u}(\cdot)$, for any given τ λ_H must be closer to λ_{CS} than λ_M.

Finally, comparing the FOCs for τ in the two modes, τ_H^* is determined by

$$\frac{G\left(\tilde{u}\left(\lambda_H\right)\right)}{g\left(\tilde{u}\left(\lambda_H\right)\right)}\left(v(\lambda_H)+v'(\lambda_H)\tau\frac{\partial\lambda_H}{\partial\tau}\right)+\left(A+\tau v(\lambda_H)\right)\frac{\partial\tilde{u}}{\partial\lambda_i}\frac{\partial\lambda_H}{\partial\tau}=0.$$

For the pure membership platform

$$\frac{G\left(\tilde{u}_0\left(\lambda_M\right)\right)}{g\left(\tilde{u}_0\left(\lambda_M\right)\right)}\left(v(\lambda_M)+v'(\lambda_M)\tau\frac{\partial\lambda_M}{\partial\tau}\right)+\left(\tau v(\lambda_M)\right)\frac{\partial\tilde{u}_0}{\partial\lambda_i}\frac{\partial\lambda_M}{\partial\tau}=0.$$

The first FOC places more weight on the extensive margin, and therefore by a similar argument as for Proposition 2 the hybrid platform will set τ to create higher $\tilde{u}(\lambda)$ than will a membership platform so long as A is sufficiently large. ■

Thus, the consumer surplus implications of going hybrid mode depends which pure mode the platform is starting at. From here on we assume that A is sufficiently large such that both parts of Corollary C.1 hold.

Moving to aggregate creator surplus (i.e., producer surplus), the implications for creator surplus are less clear-cut. Nonetheless, we have the following sufficiency results:

Corollary C.2. *In the benchmark monopoly model:*

1. *A shift from pure discovery mode to hybrid mode increases creator surplus if $G(\cdot)$ is sufficiently inelastic ($PS_H \geq PS_D$) or s sufficiently small.;*
2. *There exists a bound $\underline{U} < 0$ such that if $\min u'(\lambda) > \underline{U}$ and n_0 is not too large then a shift from pure membership mode to hybrid mode decreases creator surplus ($PS_H < PS_M$) if $G(\cdot)$ is sufficiently inelastic. If $G(\cdot)$ is sufficiently elastic and/or n_0 is sufficiently large then the shift increases producer surplus ($PS_H \geq PS_M$).*

Proof. Creator surplus in the various modes is given by the following equations

$$\begin{aligned} PS_D &= G(\tilde{u}(\lambda_D^*))a \\ PS_H &= G(\tilde{u}(\lambda_H^*))(a+(1-\tau_H^*)v(\lambda_H^*)) \\ PS_M &= G(\tilde{u}_0(\lambda_M^*))(a+(1-\tau_M^*)v(\lambda_M^*)) \end{aligned}$$

Clearly if $\lambda_{CS} \leq \lambda^*(\bar{r},1)$ $PS_D \leq PS_H$ because $\lambda_D^* = \lambda^*(\bar{r},1) \geq \lambda_H^*$ which implies $G(\tilde{u}(\lambda_D^*)) \leq G(\tilde{u}(\lambda_H^*))$ and it is also true that $a+(1-\tau_H^*)v(\lambda^*(r_H^*,\tau_H^*)) > a$. λ_{CS} is increasing in s, therefore $PS_H > PS_D$ if s is sufficiently small. However, if $\lambda_{CS} \geq \lambda^*(\bar{r},1)$ then $G(\tilde{u}(\lambda_D^*)) \geq G(\tilde{u}(\lambda_H^*))$ and the surplus comparison is in general ambiguous. However, if $G(\cdot)$ is sufficiently inelastic then the additional revenue source will outweigh any reduction in consumer participation and producers will be better off with a hybrid platform.

To prove the second result of Corollary C.2, once again compare the commission FOCs.

Hybrid:

$$\frac{G\left(\tilde{u}\left(\lambda_H\right)\right)}{g\left(\tilde{u}\left(\lambda_H\right)\right)}\left(v(\lambda_H)+v'(\lambda_H)\tau\frac{\partial\lambda_H}{\partial\tau}\right)+\left(A+\tau v(\lambda_H)\right)\frac{\partial\tilde{u}}{\partial\lambda_i}\frac{\partial\lambda_H}{\partial\tau}=0.$$

Pure membership:

$$\frac{G\left(\tilde{u}_0\left(\lambda_M\right)\right)}{g\left(\tilde{u}_0\left(\lambda_M\right)\right)}\left(v(\lambda_M)+v'(\lambda_M)\tau\frac{\partial\lambda_M}{\partial\tau}\right)+\left(\tau v(\lambda_M)\right)\frac{\partial\tilde{u}_0}{\partial\lambda_i}\frac{\partial\lambda_M}{\partial\tau}=0.$$

If $G(\cdot)$ is sufficiently inelastic, and s is not too large then the first term will dominate in both FOCs, and can only be equal to 0 if $\left(v(\lambda)+v'(\lambda)\tau\frac{\partial\lambda}{\partial\tau}\right)\approx 0$, so any differences in commission come from the differing responses of λ to τ. To facilitate this comparison, we compute

$$\frac{\partial\lambda}{\partial\tau} = \frac{-v(\lambda)}{(1-\tau)v'(\lambda)\left(\lambda^{-1} + \frac{1}{1-r}\frac{D'}{D}\frac{\partial\tilde{u}}{\partial\lambda}\right) + (a+(1-\tau)v(\lambda))\left[-\lambda^{-2} + \frac{1}{1-r}\left(\frac{\partial}{\partial\lambda}\frac{D'}{D}\frac{\partial\tilde{u}}{\partial\lambda} + \frac{D'}{D}\frac{\partial^2\tilde{u}}{\partial^2\lambda}\right)\right]}$$

From concavity of $\tilde{u}(\lambda)$, log concavity of D, and $v'(\lambda) < 0$, every term in the denominator is negative so long as $\frac{\partial\tilde{u}}{\partial\lambda} > 0$, so the negative in the numerator cancels and $\frac{\partial\lambda}{\partial\tau} > 0$. Now compare to the derivative with no discovery portal:

$$\frac{\partial\lambda}{\partial\tau} = \frac{-v(\lambda)}{(1-\tau)v\lambda'^{-1} - \lambda^{-2}(a+(1-\tau)v(\lambda))}$$

Again because of concavity of $\tilde{u}(\lambda)$ and the other conditions from above, eliminating recommendations from creators incentives means that $\frac{\partial\lambda}{\partial\tau}$ is more positive under pure membership than on a hybrid platform so long as $\frac{\partial\tilde{u}}{\partial\lambda} > 0$. This comparison will remain true if $\frac{\partial\tilde{u}}{\partial\lambda} < 0$ so long as it does not overwhelm the other two terms in the derivative, we can put a lower bound on $\frac{\partial\tilde{u}}{\partial\lambda}$ by bounding $\min u'(\lambda)$, therefore for some bound $\underline{U}$ $\frac{\partial\lambda}{\partial\tau}$ is greater under pure membership than with a hybrid platform so long as $\min u'(\lambda) > \underline{U}$. As every term except $\frac{\partial\lambda}{\partial\tau}$ in $\left(v(\lambda) + v'(\lambda)\tau\frac{\partial\lambda}{\partial\tau}\right)$ is the same between the two business modes, the platform takes a greater commission under a hybrid business model and $G(\cdot)$ inelastic further implies that consumer participation is approximately the same between the two modes, so $PS_M > PS_H$.

For n_0 large or $G(\cdot)$ highly elastic, it can be the case that $G(\tilde{u}(\lambda^*_H))a > G(\tilde{u}_0(\lambda^*_M))(a+(1-\tau^*_M)v(\lambda^*_M))$, in which case creators are always better off in the hybrid regime even if $\tau_H = 1$. ■

Consider first the shift from pure discovery mode to hybrid mode. On the one hand, the shift benefits creators by enabling them to get revenue from exclusive content. On the other hand, for large s the associated decrease in broadness $\lambda_H \leq \lambda_D$ reduces consumer participation, and the additional revenue does not compensate for that if τ is large enough. If $G(\cdot)$ is inelastic then the revenue effect dominates and if s is small so that consumer surplus increases with a hybrid platform then consumer participation increases and the hybrid mode is unambiguously better for creators.

When a pure membership platform adds a discovery portal, this attracts additional consumers, but the chase the algorithm effect means that creators may respond less to a change in τ, which encourages the platform to charge a higher commission. If $\tilde{u}'(\lambda)$ is highly negative then the chase the algorithm effect creates an increase in consumer participation *and* increases $v(\lambda)$, which can lead to creators being *more* responsive to a change in τ. In all other cases however the effect of the business model choice on creator surplus is weighing the increase in commission against the additional consumer participation.

To analyze the total surplus (W), we focus on the case with inelastic $G(\cdot)$, as in the main text. We recall from the main text that $W_H > W_D$ if and only if

$$\tilde{u}(\lambda_H) + v(\lambda_H) > \tilde{u}(\lambda_D)$$

If s is sufficiently large, then $W_H \leq W_D$. This is because $\lambda_H \leq \lambda_D$ implies that consumer welfare is reduced with the shift from pure discovery to hybrid, and if it is large enough then $\tilde{u}(\lambda_H) - \tilde{u}(\lambda_D) > v(\lambda_H)$. On the other hand, if s is quite small such that such that $\lambda_{CS} < \lambda_D$, then $\lambda_H \leq \lambda_D$ implies $W_H > W_D$.

Similarly, $W_H > W_M$ if and only if

$$\tilde{u}(\lambda_H) + v(\lambda_H) + A - C > \tilde{u}_0(\lambda_M) + v(\lambda_M)$$

Given our assumption that both parts of Corollary 3 hold, $\tilde{u}(\lambda_H) \geq \tilde{u}_0(\lambda_M)$. Therefore, $W_H \leq W_M$ can possibly occur only when $v(\lambda_M) > v(\lambda_H)$ by a large margin.

C.1.1 Social Planner

In the main text, our total surplus ranking of business models is defined by comparing the equilibrium welfare (total surplus) in each mode, assuming the platform is profit-maximizing. There, we say that misalignment occurs when the mode with the highest equilibrium profit is different from the mode with the highest equilibrium total surplus.

An alternative approach, which we consider here, is to define misalignment as occurring whenever the highest equilibrium profit is different from the mode with that would be chosen by a platform that maximizes the total surplus (i.e., a social planner or a first-best solution). Such a social planner would have exactly the same business model choices (whether to operate a discovery and/or membership portal) and the strategic variable choices (r and τ).[37]

□ **Welfare-maximizing planner**. Define $\mathbb{L}_D$, $\mathbb{L}_M$, and $\mathbb{L}_H$ as the feasible sets of λ that the planner can induce using r and/or τ under each mode. For each given mode, we can state a planner's problem as choosing λ from each feasible set. Then, the planner's problem is written as:

$$\max_{\lambda \in \mathbb{L}_{\mathbb{P}_l}, P_l \in \{D,M,H\}} G\left(\tilde{u}_0(\lambda) + (1 - I_{\{M\}})\tilde{u}(\lambda)\right)\left(I_{\{M\}}\tilde{u}_0(\lambda) + (1 - I_{\{M\}})\left[\tilde{u}(\lambda) + A\right] + a + v(\lambda) \cdot I_{\{not\ D\}}\right)$$
$$+ \int_{(1-I_{\{M\}})\tilde{u}(\lambda)+I_{\{M\}}\tilde{u}_0(\lambda)}^{\infty} xg(x)dx - (1 - I_{\{M\}})C$$

where the decision variable P_l represents the choice of business model of the planner.

The planner will never choose $P_l = D$ because the equilibrium replication argument from Proposition 3 implies that it is worse than $P_l = H$. It remains to compare $P_l = H$ and $P_l = M$. In what follows, we assume $G(\cdot)$ is highly inelastic.

Consider $P_l = H$, the FOC with respect to $\lambda \in \mathbb{L}_H$

$$G\left(\tilde{u}(\lambda)\right)\left[\tilde{u}'(\lambda) + v'(\lambda)\right] + g\left(\tilde{u}(\lambda)\right)\tilde{u}'(\lambda)\left[(1 - I_{\{M\}})\left[A - C\right] + a + v(\lambda)\right] = 0 \tag{C.2}$$

Given $G(\cdot)$ is highly inelastic, then the FOC reduces to

$$\tilde{u}'(\lambda) + v'(\lambda) = 0, \tag{C.3}$$

and we denote the welfare-maximizing solution in hybrid mode as $\lambda_H^W \in \mathbb{L}_H$. Likewise, consider $P_l = M$. By the same analysis, the corresponding FOC is

$$\tilde{u_0}'(\lambda) + v'(\lambda) = 0, \tag{C.4}$$

and we denote the welfare-maximizing solution in this mode as $\lambda_M^W \in \mathbb{L}_M$. Therefore, maximizing welfare in each case is simply a combination of maximizing consumer participation utility and transaction revenue. Because $v'(\lambda) < 0$ it is immediate that $\lambda_H^W \leq \lambda_{CS}$ whenever $\lambda_{CS} \in \mathbb{L}_H$, where λ_{CS} is defined in (C.1) (likewise, $\lambda_M^W \leq \lambda_{CS}$ whenever $\lambda_{CS} \in \mathbb{L}_M$). Moreover, the inequality $\tilde{u_0}'(\lambda) \geq \tilde{u}'(\lambda)$ always holds for any given λ, and so the FOCs mean that $\lambda_H^W \leq \lambda_M^W$ whenever $\lambda_M^W \in \mathbb{L}_H$.[38]

We now identify a sufficient condition for $P_l = H$ to be the optimal mode for the planner, as compared to $P_l = M$. From the social planner's perspective, the upsides of introducing a discovery portal are the advertising revenue $A - C$ and the reduction in the effective search cost paid by consumers, whereas

[37]Some might consider consumer surplus as an appropriate objective for the social planner. It is trivial to show that such a planner would weakly prefer choosing the hybrid mode over the other modes using a outcome-replication argument as in Proposition 3. Hence, here we focus our attention on total surplus.

[38]Allowing for a change in consumer participation increases the relative importance of $\tilde{u}(\lambda)$ relative to $v(\lambda)$. $g(\cdot)$ and the term in square brackets in the second term are both positive, so the second term has the same sign as $\tilde{u}'(\lambda)$ and leads the planner to set λ slightly higher than it would if it were not taking participation changes into account.

the downside is that the chase-the-algorithm effect result in the difference in the maximization domain $\mathbb{L}_H \neq \mathbb{L}_M$. By the replication argument, a sufficient for $P_l = H$ to be the optimal mode is

$$\lambda_M^W \in \mathbb{L}_H.$$

This condition fails only if the the chase-the-algorithm effect is so severe such that $\lambda_M^W \notin \mathbb{L}_H$.

In sum, consistent with Figure 3, the discussion suggests that a hybrid mode tends to be preferred over a pure membership mode for the planner. The only exception is the arguably rare case in which all of the following conditions hold simultaneously:

- $A - C$ is small enough.
- Search cost s is small enough such that the reduction in search cost from the discovery portal is small.
- $v(\lambda)$ decreases quickly

□ **Misalignment of business mode adoption of a profit-maximizing platform**. For simplicity, suppose the parameters are such that the hybrid mode $P_l = H$ is the optimal mode for the planner (see the sufficient condition above). Then, from Proposition 2, it is immediate that misalignment occurs whenever the profit-maximizing platform adopts the pure membership mode, i.e., whenever $A < A_{mono}$.

□ **Misalignment in the induced design for a given mode**. For the pure discovery mode, it is clear that $\lambda_D^W = \lambda_D^*$. Meanwhile, the comparisons in each of the other modes are ambiguous in general. To see this, consider the hybrid mode. If A is large, then the profit-maximizing platform will likely induce $\lambda_H^* > \lambda_H^W$ because it only captures the fraction $\tau \leq 1$ of $v(\lambda)$, and so it will underweight exclusive content revenue relative to consumer participation unless $\tau = 1$. However if A is relatively small then the platform places more weight on $v(\lambda)$ and may even set $\lambda_H^* < \lambda_H^W$ because the platform does not take $\tilde{u}(\lambda)$ into account except as far as it affects participation.

D Extension: endogenous ad and revenue instruments

We denote $\lambda^* = \lambda^*(r, \tau)$ as the solution to the fixed-point equation

$$\lambda^*(p, f, r, \tau) = \arg \max_{\lambda_i \in [0,1]} \left\{ D\left(\frac{\tilde{u}(\lambda_i)}{1-r}; \frac{\tilde{u}(\lambda^*)}{1-r}\right) (a + (1-f)pA + (1-\tau)v(\lambda_i)) \lambda_i \right\}.$$

Obviously, $\lambda^* > 0$; whereas $\partial\lambda^*/\partial\tau \geq 0$, $\partial\lambda^*/\partial f \leq 0$, $\partial\lambda^*/\partial p \geq 0$, $\partial\lambda^*/\partial A \geq 0$ (with the inequality strict whenever $\lambda^* \in (0,1)$); whereas $\partial\lambda^*/\partial r$ has the same sign as $\partial\tilde{u}/\partial\lambda_i$, meaning that $\tilde{u}(\lambda^*(p, f, r, \tau))$ is always increasing in r.

Notice $\lambda_H^* = \lambda^*(p, f, r, \tau)$, $\lambda_D^* = \lambda^*(p, f, r, 1)$, and $\lambda_M^* = \lambda^*(0, 1, -\infty, \tau)$. Platform profits are

$$\begin{aligned}
\Pi_H(p, f, r, \tau) &= G(\tilde{u}(\lambda^*(p, f, r, \tau)) - p)(fpA + \tau v(\lambda^*(p, f, r, \tau))) - C \\
\Pi_D(p, f, r) &= G(\tilde{u}(\lambda^*(p, f, r, 1)) - p) fpA - C \\
\Pi_M(\tau) &= G(\tilde{u}_0(\lambda^*(0, 1, -\infty, \tau))) \tau v(\lambda^*(0, 1, -\infty, \tau)).
\end{aligned}$$

where recall $\tilde{u}(\lambda_i) \equiv u(\lambda_i) - \frac{s}{\lambda_i}$ and $\tilde{u}_0(\lambda_i) \equiv u(\lambda_i) - \frac{n_0 s}{\lambda_i}$. Denote $(p_k^*, f_k^*, r_k^*, \tau_k^*)$ as the optimal platform choices in each mode $k \in \{D, M, H\}$, whenever the choices are applicable.

Proposition D.1. *Consider a move from the pure discovery mode to the hybrid mode. Then, the platform profit increases ($\Pi_H^* > \Pi_D^*$); Suppose $\bar{r} \to 1$, then the equilibrium content design becomes less broad ($\lambda_H^* \leq \lambda_D^*$), and $f_D^* = f_H^* = 1$.*

Proof. We apply the profit replication argument:

$$\begin{aligned}
\Pi_D^* &= G\left(\tilde{u}\left(\lambda^*(p_D^*, f_D^*, r_D^*, 1)\right) - p_D^*\right) f_D^* p_D^* A - C \\
&< G\left(\tilde{u}\left(\lambda^*(p_D^*, f_D^*, r_D^*, 1)\right) - p_D^*\right)\left(f_D^* p_D^* A + \tau v\left(\lambda^*(p_D^*, f_D^*, r_D^*, 1)\right)\right) - C \\
&= \Pi_H(p_D^*, f_D^*, r_D^*, 1) \\
&\leq \Pi_H^*
\end{aligned}$$

For the result on the equilibrium design, we first note $r_D^* = \bar{r}$ because $\tilde{u}(\lambda^*(p, f, r, \tau))$ is always increasing in r. Then, if $r_D^* = \bar{r} \to 1$, then each creator i chooses λ_i to maximize $\tilde{u}(\lambda_i)$ in order to get recommended akin to a homogeneous-good Bertrand competition, so

$$\lambda_D^* = \lambda_{CS}^* \equiv \arg\max_{\lambda_i} \tilde{u}(\lambda_i) = \arg\max_{\lambda_i} \left\{ u(\lambda_i) - \frac{s}{\lambda_i} \right\}.$$

If $r_H^* = \bar{r} \to 1$, then $\lambda_H^* = \lambda_{CS}$ and we are done. Consider $r_H^* < \bar{r}$ and suppose by contradiction $\lambda_H^* = \lambda(p_H^*, f_H^*, r_H^*, \tau_H^*) > \lambda_D^* = \lambda_{CS}$. By strict concavity of $\tilde{u}$, this means that

$$\frac{\partial \tilde{u}}{\partial \lambda_i}|_{\lambda_H^*} < 0.$$

Consider a deviation by the hybrid mode platform to sensitivity $r_H^* + \epsilon$, which is always feasible given $r_H^* < \bar{r}$. Then, inequality $\frac{\partial \tilde{u}}{\partial \lambda_i}|_{\lambda_H^*} < 0$ implies $\lambda_H^* > \lambda^*(p_H^*, f_H^*, r_H^* + \epsilon, \tau_H^*)$ and $\tilde{u}(\lambda_H^*) < \tilde{u}(\lambda(p_H^*, f_H^*, r_H^* + \epsilon, \tau_H^*))$ because $\partial\lambda^*/\partial r$ has the same sign as $\partial\tilde{u}/\partial\lambda_i$. Then, given $v(.)$ is decreasing, we get

$$\begin{aligned}
\Pi_H^* &= G\left(\tilde{u}\left(\lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)\right) - p_H^*\right)\left(f_H^* p_H^* A + \tau_H^* v\left(\lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)\right)\right) - C \\
&< G\left(\tilde{u}(\lambda^*(p_H^*, f_H^*, r_H^* + \epsilon, \tau_H^*)) - p_H^*\right)\left(f_H^* p_H^* A + v\left(\lambda^*(p_H^*, f_H^*, r_H^* + \epsilon, \tau_H^*)\right)\right) - C \\
&= \Pi_H(p_H^*, f_H^*, r_H^* + \epsilon, \tau_H^*),
\end{aligned}$$

a contradiction to optimality of r_H^*. Hence, we conclude $\lambda_H^* \leq \lambda_D^*$.

For the result on the equilibrium fees, we first note $\bar{r} \to 1$ implies the pure discovery mode maximization problem is

$$\max_{p,f} G\left(\tilde{u}\left(\lambda_{CS}\right) - p\right) f p A - C,$$

which is monotone in f and so $f_D^* = 1$. If $\lambda_H^* = \lambda_D^* = \lambda_{CS}$, then the same argument applies to the hybrid mode so that $f_H^* = 1$. Consider $\lambda_H^* < \lambda_{CS}$, which means $\frac{\partial \tilde{u}}{\partial \lambda_i}|_{\lambda_H^*} > 0$ by strict concavity of $\tilde{u}$. By contradiction, suppose $f_H^* < 1$, the platform can increase r and f simultaneously to make λ^* constant (recall $\partial\lambda^*/\partial f < 0$ whereas $\partial\lambda^*/\partial r$ has the same sign as $\partial\tilde{u}/\partial\lambda_i > 0$) and strictly increase its revenue $fpA + \tau v(\lambda^*)$, a contradiction. ■

Before proving the next result, we make the following observations:

Lemma D.1. *Suppose $s > \bar{s}_{max}$. When maximizing $\Pi_H(p, f, r, \tau)$, the boundary constraints $f \leq 1$,. $r \in [\underline{r}, \bar{r}]$, and $\tau \leq 1$ have the following properties:*

1. Either $f_H^* = 1$ or $\tau_H^* = 1$ (or both);
2. Either $r_H^* = \underline{r}$ or $\tau_H^* = 1$ (or both).

Proof. Denote the overall equilibrium design at $\lambda_H^* = \lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)$. If $\lambda_H^* = 1$, then $\tau_H^* < 1$ is obviously sub-optimal because

$$\Pi_H(p_H^*, f_H^*, r_H^*, \tau_H^*) = G\left(\tilde{u}\left(1\right) - p_H^*\right)\left(f_H^* p_H^* A + \tau_H^* v\left(1\right)\right) - C$$

can be strictly increased by raising commission to $\tau_H^* + \epsilon$. Consider $\lambda_H^* < 1$, in what follows.

(1) By contradiction, suppose $f_H^* < 1$ and $\tau_H^* < 1$. The platform can increase τ and f simultaneously to make λ^* constant while strictly increasing its revenue $fpA + \tau v(\lambda^*)$.

(2) By contradiction, suppose $r_H^* > \underline{r}$ and $\tau_H^* < 1$. By the supposition and strict concavity of $\tilde{u}$, we have $\frac{\partial \tilde{u}}{\partial \lambda_i}|_{\lambda_H^*} > 0$. Then, the platform can increase τ and lowering r simultaneously to make λ^* constant while strictly increasing its revenue $fpA + \tau v(\lambda^*)$. ■

Proposition D.2. *Consider a move from the pure membership mode to the hybrid mode and suppose* $s > \bar{s}_{max}$. *There exist weakly positive thresholds* A_{mono}, A'_{mono}, *such that:*

- *The equilibrium content design becomes broader* ($\lambda_H^* \geq \lambda_M^*$) *if* $A \geq A'_{mono}$*;*
- *The platform profit increases if and only if* $A \geq A_{mono}$*; moreover,* $A_{mono} > 0$ *if* $\tau_M^* < 1$ *and* G *is sufficiently inelastic.*

Proof. We first prove that Π_H^* is monotone increasing in A (note that $\partial\lambda^*/\partial A \geq 0$ so this involves additional steps compared to Proposition 2). Starting from any arbitrary $A = A_1$, consider a small increase to $A_2 > A_1$. Denote the maximizer at $A = A_1$ as $(p_H^*, f_H^*, r_H^*, \tau_H^*)$. If $f_H^* = 1$ at $A = A_1$ then $\lambda^*(p, f_H^*, r, \tau)$ is independent of A so that $\Pi_H^*(A_2) \geq \Pi_H^*(A_1)$ by envelope theorem. If $f_H^* < 1$ at $A = A_1$, then let

$$f'_H = 1 - (1 - f_H^*)\frac{A_1}{A_2} > f_H^*$$

so that

$$\lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)|_{A=A_1} = \lambda^*(p_H^*, f'_H, r_H^*, \tau_H^*)|_{A=A_2}.$$

Then

$$\begin{aligned}
\Pi_H^*|_{A=A_2} &\geq G\left(\tilde{u}\left(\lambda^*(p_H^*, f'_H, r_H^*, \tau_H^*)|_{A=A_2}\right) - p\right)\left(fp_H^* A_2 + \tau v\left(\lambda^*(p_H^*, f'_H, r_H^*, \tau_H^*)|_{A=A_2}\right)\right) - C \\
&= G\left(\tilde{u}\left(\lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)|_{A=A_1}\right) - p\right)\left(fp_H^* A_2 + \tau v\left(\lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)|_{A=A_1}\right)\right) - C \\
&> G\left(\tilde{u}\left(\lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)|_{A=A_1}\right) - p\right)\left(fp_H^* A_1 + \tau v\left(\lambda^*(p_H^*, f_H^*, r_H^*, \tau_H^*)|_{A=A_1}\right)\right) - C \\
&= \Pi_H^*|_{A=A_1}
\end{aligned}$$

where the first inequality uses the definition of $\Pi_H^*(A_2)$ being the maximum, the equality uses the definition of f'_H above, and the last inequality uses $A_2 > A_1$. Hence, we conclude Π_H^* is monotone increasing in A. The intermediate value theorem and implicit function theorem together prove the existence and uniqueness of threshold $A_{mono} \geq 0$ (if $\Pi_H^* > \Pi_M^*$ for all $A \geq 0$, then we set $A_{mono} = 0$).

To show $A_{mono} > 0$ if G is sufficiently inelastic, let $A = 0$ and $G(.)$ be a constant, then

$$\begin{aligned}
\Pi_H^* &= G \times \tau_H^* v(\lambda^*(r_H^*, \tau_H^*)) - C \\
&< G \times \tau_H^* v(\lambda^*(-\infty, \tau_H^*)) - C \\
&\leq G \times \tau_M^* v(\lambda^*(-\infty, \tau_M^*)) = \Pi_M^*.
\end{aligned}$$

where the first inequality follows from $v(.)$ being decreasing and $\partial\lambda^*/\partial r > 0$ (implied by $\tilde{u}(\lambda_i)$ being monotone increasing) and $\lambda^*(-\infty, \tau_H^*) < 1$ (implied by $\tau_M^* < 1$), and the second inequality follows from the definition of τ_M^* and $C \geq 0$.

To prove the results on the equilibrium design, we first note that if $\tau_H^* = 1$, then

$$\begin{aligned}
\lambda_H^* &= \lambda^*(p_H^*, f_H^*, r_H^*, 1) \\
&\geq \lambda^*(0, 1, -\infty, 1) \qquad (\lambda^* \text{ increasing in } r \geq 0 \text{ and } (1-f)p \geq 0) \\
&\geq \lambda^*(0, 1, -\infty, \tau_M^*) \quad (\lambda^* \text{ increasing in } \tau \leq 1) \\
&= \lambda_M^*.
\end{aligned}$$

If instead $\tau_H^* < 1$, then it implies $f_H^* = 1$ and $r_H^* = \underline{r}$ (Lemma D.1). With a slight abuse of notation, denote in what follows

$$\lambda^*(r, \tau) = \lambda^*(p, 1, r, \tau)$$

where we note $f = 1$ means $\lambda^*(p, 1, r, \tau)$ becomes independent of p. This is exactly $\lambda^*(r, \tau)$ in the proof of Proposition 2. Using this notation, we can simplify the optimal choices in the hybrid mode as

$$(p_H^*, \tau_H^*) = \arg\max_{p,\tau} \left\{ G\left(\tilde{u}(\lambda^*(\underline{r}, \tau)) - p\right)\left(pA + \tau v(\lambda^*(\underline{r}, \tau))\right) \right\}.$$

If we can show that τ_H^* is increasing in A, then it follows that there exists a threshold A'_{mono} such that $\tau_H^* \geq \tau_M^*$ for all $A \geq A'_{mono}$. Then, we are done because $A \geq A'_{mono}$ then implies

$$\lambda_H^* = \lambda^*(\underline{r}, \tau_H^*) \geq \lambda^*(-\infty, \tau_H^*) \geq \lambda^*(-\infty, \tau_M^*) = \lambda_M^*.$$

To show τ_H^* is increasing in A, note that this is obvious if either τ_H^* or p_H^* is non-interior. If instead τ_H^* and p_H^* are interior solutions, they are jointly pinned down by FOC that can be simplified as

$$\begin{aligned}
v(\lambda^*) + \left(A\frac{\partial \tilde{u}}{\partial \lambda_i} + \tau_H^* v'(\lambda^*)\right)\frac{\partial \lambda^*}{\partial \tau} &= 0 \\
A - (pA + \tau_H^* v(\lambda^*(r, \tau)))\frac{g\left(\tilde{u}(\lambda^*) - p\right)}{G\left(\tilde{u}(\lambda^*) - p\right)} &= 0.
\end{aligned}$$

where $\lambda^* = \lambda^*(\underline{r}, \tau_H^*)$. Total differentiation on the first equation gives

$$\left[2v'(\lambda^*)\frac{\partial \lambda^*}{\partial \tau} + \left(A\frac{\partial \tilde{u}}{\partial \lambda_i} + \tau_H^* v'(\lambda^*)\right)\frac{\partial^2 \lambda^*}{\partial \tau^2} + \left(A\frac{\partial^2 \tilde{u}}{\partial \lambda_i^{*2}} + \tau_H^* v''(\lambda^*)\right)\left(\frac{\partial \lambda^*}{\partial \tau}\right)^2\right]\frac{d\tau_H^*}{dA} = -\frac{\partial \tilde{u}}{\partial \lambda_i}\frac{\partial \lambda^*}{\partial \tau}$$

$$\Longrightarrow \frac{d\tau_H^*}{dA} = \frac{-\frac{\partial \tilde{u}}{\partial \lambda_i}\frac{\partial \lambda^*}{\partial \tau}}{2v'(\lambda^*)\frac{\partial \lambda^*}{\partial \tau} - \frac{v(\lambda^*)}{\frac{\partial \lambda^*}{\partial \tau}}\frac{\partial^2 \lambda^*}{\partial \tau^2} + \left(A\frac{\partial^2 \tilde{u}}{\partial \lambda_i^2} + \tau_H^* v''(\lambda^*)\right)\left(\frac{\partial \lambda^*}{\partial \tau}\right)^2} \geq 0$$

where the denominator is positive by the quasiconcavity condition. ■

E Extension: asymmetric creators

E.1 Summary

In this section, we extend the monopoly model by introducing asymmetric creators who are heterogeneous in their flat per-viewer gain $a_i \in [\underline{a}, \bar{a}]$ in (3), and denote the corresponding distribution function as F. That is, creators are heterogeneous in terms of the profitability of their individual sponsorship revenues or intrinsic image gain. In line with the baseline model, we assume each creator's per-viewer gain a_i and its type t_i are independently distributed.

Each creator i will make asymmetric content design decisions in the equilibrium, denoted as $\lambda(a_i)$. Note that we recover the baseline model if $\underline{a} = \bar{a}$. We remind the reader here that we are assuming $s \geq \bar{s}_{\max}$ holds so that $\tilde{u}(\lambda_i)$ is monotone increasing.

One complication in this asymmetric environment is that we need to work with the *distribution* of equilibrium design rather than a single symmetric equilibrium design. To ensure that the analysis remains tractable, we impose the logit recommendation function in (5). We assume that the parameters are such that $\lambda(a_i) \in (0,1)$ for all $a_i \in [\underline{a}, \bar{a}]$, that F is uniform, and that the linear specification in (4) holds.

Consider the creator-consumer subgame for any arbitrarily given r and τ (recall that $\tau = 1$ in the pure discovery mode, and $r = -\infty$ in the pure membership mode). Following Section 4, in the equilibrium of the subgame, each creator i joins both portals of the platform and sets

$$\lambda(a_i) = \arg\max_{\lambda_i \in [0,1]} \left\{ \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) \times \lambda_i \times (a_i + (1-\tau) v(\lambda_i)) \right\}.$$

Notice that $\lambda^*(a_i)$ is increasing in a_i: creators with a higher per-viewer gain opt for broader designs than lower-type creators. Then, define $\tilde{F}$ as the *effective (i.e., recommendation-weighted) distribution of content design*. The corresponding CDF is

$$\tilde{F}(z) = \frac{\int_{\lambda(\underline{a})}^{z} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) dF_\lambda(\lambda_i)}{\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) dF_\lambda(\lambda_i)} \quad \text{for} \quad z \in [\lambda(\underline{a}), \lambda(\bar{a})],$$

where $F_\lambda(\lambda_i) = \Pr(\lambda(a_i) \le \lambda_i) = F(\lambda^{-1}(\lambda_i))$ is the unconditional (i.e., not weighted by recommendations) distribution of content design $\lambda(a_i)$. In other words, $\tilde{F}$ is the distribution of creator design that a consumer faces when searching on the platform and following the recommendation. Notice that if $r = -\infty$, then $\tilde{F} = F_\lambda$.

In this environment, consumers strictly prefer following the platform's recommendation in each step of the search because the recommendation rule (2) is consistent with the Pandora's rule. In other words, consumers face a more favorable distribution of search reservation value when they follow the recommendation than not following: distribution $\tilde{F}$ is higher than F_λ in the sense of first-order stochastic dominance (FOSD), holding the decisions of the creators fixed. Moreover, even though λ_i are asymmetric in the equilibrium, an application of Weitzman's (1979) stopping rule shows that consumers indeed stop searching upon observing a positive match value with a creator, as in Lemma 1. Each consumer believes that the distribution of λ_i is given by $\tilde{F}$, and initiates search if and only if outside option satisfies $x \le \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z)$.

The following result is analogous to Proposition 1 in the baseline model, showing how the platform's choices affect the effective distribution of the content design.

Lemma E.1. *In the model of asymmetric creators described in this section, the recommendation-weighted distribution of content design $\tilde{F}(z; r, \tau)$ in (E.3) is increasing in τ and r in the sense of first-order stochastic dominance (FOSD).*

Based on this result, we verify that the qualitative insights in Propositions 2 and 3 continue to hold. The only mechanical difference is that, when creators make asymmetric content design decisions in the equilibrium, the platform's choice of mode now influences the market outcome through an additional *recommendation-shifting effect* where by a higher recommendation sensitivity r or a higher commission τ means that creators with a higher a_i are more likely to be recommended.

E.2 Analytical details and proofs

Creators are asymmetric in their intrinsic image gain $a_i \in [\underline{a}, \bar{a}]$, following distribution F, which is assumed to be uniform. The recommendation function follows the logit specification (5) so that $\frac{D'(z)}{D(z)} = 1$ due to the continuum assumption. As stated in the main text, we impose the linear microfoundation in (4):

$$v(\lambda_i) = (v_0 - \lambda_i)\gamma \qquad \text{and} \qquad u(\lambda_i) = b + (v_0 - \lambda_i)(1-\gamma).$$

Recall we focus on the case of sufficiently large search cost $s \geq 1-\gamma$ so that $\tilde{u}(\lambda_i) = b + (v_0 - \lambda_i)(1-\gamma) + \frac{s}{\lambda_i}$ is monotone increasing. Recall

$$\lambda(a_i) = \arg\max_{\lambda_i \in [0,1]} \left\{ \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) \times \lambda_i \times (a_i + (1-\tau)v(\lambda_i)) \right\}.$$

We assume the parameters are such that $\lambda(a_i) \in (0,1)$ for all $r \in [\underline{r}, \bar{r}]$ and $\tau \leq \max\{\tau_M^*, \tau_H^*\}$ such that it is always pinned down by first-order condition:

$$0 = \frac{1}{\lambda} + \frac{1}{1-r}\left(\frac{s}{\lambda^2} - (1-\gamma)\right) - \frac{(1-\tau)\gamma}{a + (1-\tau)(v_0 - \lambda)\gamma} \tag{E.1}$$

Then, observe that $\lambda(a_i) = \lambda(a_i; r, \tau)$ is strictly increasing in τ, r, and a_i. In particular, its inverse $\lambda^{-1} : [\lambda(\underline{a}), \lambda(\bar{a})] \to [\underline{a}, \bar{a}]$ is well-defined by inverting (E.1):

$$\begin{aligned} \lambda^{-1}(\tilde{\lambda}_i) &= \left(-v'(\lambda_i)\left(\frac{1}{\lambda_i} + \frac{1}{1-r}\frac{\partial \tilde{u}}{\partial \lambda_i}\right)^{-1} - v(\lambda_i)\right)(1-\tau) \\ &= \left(\frac{1}{1/\tilde{\lambda}_i + \frac{1}{1-r}\partial \tilde{u}/\partial \lambda_i} - v_0 + \tilde{\lambda}_i\right)(1-\tau)\gamma \end{aligned} \tag{E.2}$$

and clearly $\frac{\partial \lambda^{-1}(z)}{\partial z} > 0$.

Notice that, in the absence of platform recommendation, the *unconditional distribution of content design* $\lambda(a_i)$ is

$$\begin{aligned} F_\lambda(z) = \Pr(\lambda(a_i) \leq z) &= \Pr(a_i \leq \lambda^{-1}(z)) \\ &= F(\lambda^{-1}(z)) \qquad \text{for } z \in [\lambda(\underline{a}), \lambda(\bar{a})]. \end{aligned}$$

Denote

$$f_\lambda(z) = \frac{\partial}{\partial z} F(\lambda^{-1}(z)) = f(\lambda^{-1}(z))\frac{\partial \lambda^{-1}(z)}{\partial z}.$$

Then, define $\tilde{F}$ as the (recommendation-weighted) *effective distribution of content design*, where the corresponding cumulative distribution function is

$$\tilde{F}(z) = \frac{\int_{\lambda(\underline{a})}^{z} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i}{\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i} \quad \text{for} \quad z \in [\lambda(\underline{a}), \lambda(\bar{a})], \tag{E.3}$$

and note $\tilde{F}$ depends on τ and r. Notice if $r = -\infty$ then $\tilde{F}(z) = F(\lambda^{-1}(z))$. To make explicit the dependency, we will sometimes write the CDF as $\tilde{F}(z; r, \tau)$. Recall we assume F is uniform.

We now verify the claim on consumer search behavior stated in the main text:

Lemma E.2. *For given r on the equilibrium path, each consumer believes that the distribution of design is described by CDF $\tilde{F}$, and initiates search if and only if*

$$x \leq \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z)\tilde{F}(z),$$

and does so through the discovery portal (follows the recommendation in every step of search). Upon observing a positive match value with a creator, the consumer stops searching and becomes a viewer of the creator.

Proof. On the equilibrium path, suppose a consumer has inspected a creator i and the realized value is $u_{ij} = u(\lambda_i) > 0$. By Pandora's rule (Weitzman 1979) and the assumption of a continuum of creators, we know a consumer stops searching if and only if $u(\lambda_i)$ is higher than the expected incremental gain, that is,

$$u(\lambda_i) \geq \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z)\tilde{F}(z). \tag{E.4}$$

To establish (E.4), recall we are focusing on the case of sufficiently large s so that $\tilde{u}(z)$ is increasing. Therefore,

$$\begin{aligned} \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z)\tilde{F}(z) \leq \tilde{u}(\lambda(\bar{a})) \quad &< \quad u(\lambda(\bar{a})) \\ &\leq \quad u(\lambda_i) \text{ for all } \lambda_i \in [\lambda(\underline{a}), \lambda(\bar{a})] \end{aligned}$$

where the last two inequalities used $\tilde{u}(\lambda_i) = u(\lambda_i) - \frac{s}{\lambda_i}$ and $u(\lambda_i)$ being decreasing for all λ_i. Consumers strictly prefer following the platform's recommendation in each step of the search because the recommendation rule (2) is consistent with the Pandora's rule, as stated in the text. Finally, by stationarity, the ex-ante expected surplus from initiating search is exactly $\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z)\tilde{F}(z)$. Note this proof nests $r = -\infty$ as a special case. ■

Then, the following result is analogous to Proposition 1 in the main text.

Lemma E.3. *In the model of asymmetric creators described in this section, the recommendation-weighted distribution of content design $\tilde{F}(z; r, \tau)$ in (E.3) is increasing in τ and r in the sense of first-order stochastic dominance (FOSD).*

Proof. We note the distribution support $[\lambda(\underline{a}), \lambda(\bar{a})]$ shifts upward when τ and r increases (recall $\partial\lambda(a_i)/\partial\tau > 0$ and $\partial\lambda(a_i)/\partial r > 0$), so it remains to check the functional form of $\tilde{F}(z)$. Denote

$$\begin{aligned} Z(\lambda_i) \quad &\equiv \quad f_\lambda(\lambda_i) \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) \\ &= \quad f(\lambda^{-1}(z)) \frac{\partial \lambda^{-1}(z)}{\partial z} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) \end{aligned}$$

where we note $\lambda^{-1}(z)$ depends on τ and r.

Consider the comparative statics with respect to τ. Taking the derivative,

$$\begin{aligned} \frac{d\tilde{F}(z)}{d\tau} \quad = \quad & \left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} Z(\lambda_i)d\lambda_i\right)\left(\int_{\lambda(\underline{a})}^{z} \frac{\partial Z(\lambda_i)}{\partial \tau} d\lambda_i\right) - \left(\int_{\lambda(\underline{a})}^{z} Z(\lambda_i)d\lambda_i\right)\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \frac{\partial Z(\lambda_i)}{\partial \tau} d\lambda_i\right) \\ & - \left(\int_{z}^{\lambda(\bar{a})} Z(\lambda_i)d\lambda_i\right) Z(\lambda(\underline{a}))\frac{\partial \lambda(\underline{a})}{\partial \tau} - \left(\int_{\lambda(\underline{a})}^{z} Z(\lambda_i)d\lambda_i\right) Z(\lambda(\bar{a}))\frac{\partial \lambda(\bar{a})}{\partial \tau}, \end{aligned}$$

where the second line is negative. Therefore, $\frac{d\tilde{F}(z)}{d\tau} \leq 0$ if

$$\frac{\int_{\lambda(\underline{a})}^{z} \frac{\partial Z(\lambda_i)}{\partial \tau} d\lambda_i}{\int_{\lambda(\underline{a})}^{z} Z(\lambda_i)d\lambda_i} \leq \frac{\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \frac{\partial Z(\lambda_i)}{\partial \tau} d\lambda_i}{\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} Z(\lambda_i)d\lambda_i}, \tag{E.5}$$

which holds if the LHS of (E.5) is increasing in z. The corresponding derivative of the LHS of (E.5) is positive if and only if

$$\begin{aligned}\frac{\partial Z(z)/\partial\tau}{Z(z)} &\geq \frac{\int_{\lambda(\underline{a})}^{z}\frac{\partial Z(\lambda_i)}{\partial\tau}d\lambda_i}{\int_{\lambda(\underline{a})}^{z}Z(\lambda_i)d\lambda_i} \\ &= \int_{\lambda(\underline{a})}^{z}\frac{\partial Z(\lambda_i)/\partial\tau}{Z(\lambda_i)}\left(\frac{Z(\lambda_i)}{\int_{\lambda(\underline{a})}^{z}Z(\lambda_i)d\lambda_i}\right)d\lambda_i\end{aligned} \tag{E.6}$$

Hence, a sufficient condition for (E.6) is $\frac{\partial Z(z)/\partial\tau}{Z(z)}$ being increasing in z. Simplifying:

$$\begin{aligned}\frac{\partial Z(z)/\partial\tau}{Z(z)} &= \frac{\partial f_\lambda(z)/\partial\tau}{f_\lambda(\lambda_i)} \\ &= \frac{f'(\lambda^{-1}(z))}{f(\lambda^{-1}(z))}\frac{\partial\lambda^{-1}(z)}{\partial\tau}+\frac{\partial^2\lambda^{-1}(z)/\partial z\partial\tau}{\partial\lambda^{-1}(z)/\partial z}\end{aligned}$$

Imposing uniform F and (E.2), we get $f'=0$ and $\frac{\partial^2\lambda^{-1}(z)}{\partial z\partial\tau}=-\frac{\partial\lambda^{-1}(z)/\partial z}{1-\tau}$ and so

$$\frac{\partial Z(z)/\partial\tau}{Z(z)}=-\frac{1}{1-\tau}$$

which is independent of z if $f'=0$. Then, given that the support $[\lambda(\underline{a}),\lambda(\bar{a})]$ shifts upward as τ increases, it follows that $\tilde{F}(z)$ is increasing in FOSD sense when τ increases.

Consider the comparative statics with respect to r. By the same steps as previous case, we know a sufficient condition for $\frac{d\tilde{F}(z)}{dr}\leq 0$ is $\frac{\partial Z(z)/\partial r}{Z(z)}$ being increasing in z. Simplifying by imposing uniform F, we get

$$\frac{\partial Z(z)/\partial r}{Z(z)}=\frac{\partial^2\lambda^{-1}(z)/\partial z\partial r}{\partial\lambda^{-1}(z)/\partial z}+\frac{\tilde{u}(z)}{(1-r)^2}.$$

Solving for the derivatives with (E.2), we get

$$\frac{d\lambda^{-1}(z)}{dx}=\left(\frac{d\psi/dx+\psi^2}{\psi^2}\right)(1-\tau)\gamma>0$$

where $\psi=\frac{1}{z}+\frac{1}{1-r}\left(\frac{s}{z^2}-(1-\gamma)\right)>0$ and $d\psi/dx+\psi^2>0$ because $1-\gamma\leq s$. Then,

$$\frac{\partial^2\lambda^{-1}(z)/\partial z\partial r}{\partial\lambda^{-1}(z)/\partial z}=\frac{\psi\left(\frac{d\psi}{dxdr}\right)-2\left(\frac{d\psi}{dx}\right)\left(\frac{d\psi}{dr}\right)}{\psi\frac{d\psi}{dx}+\psi^3}.$$

Simplifying the algebra, it can be verified that the expression above is increasing in z. ■

The profit function of the monopoly platform in the pure membership, pure discovery, and hybrid modes are, respectively,

$$\begin{aligned}\Pi_M(\tau) &= G\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})}\tilde{u}(z)d\tilde{F}(z;-\infty,\tau)\right)\tau\int_{\lambda(\underline{a})}^{\lambda(\bar{a})}v(z)d\tilde{F}(z;-\infty,\tau); \\ \Pi_D(r) &= G\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})}\tilde{u}(z)d\tilde{F}(z;r,1)\right)A-C; \\ \Pi_H(r,\tau) &= G\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})}\tilde{u}(z)d\tilde{F}(z;r,\tau)\right)\left(A+\tau\int_{\lambda(\underline{a})}^{\lambda(\bar{a})}v(z)d\tilde{F}(z;r,\tau)\right)-C\end{aligned}$$

Denote Π_M^*, Π_D^*, and Π_H^* as the respective maximized profit, and the corresponding effective distribution of content design as $\tilde{F}_M^*$, $\tilde{F}_D^*$, and $\tilde{F}_H^*$.

We are now ready to compare across the modes of operations.

Proposition E.1. *Consider a move from the pure discovery mode to the hybrid mode. Then, in the model of asymmetric creators described in this section, the equilibrium distribution of content design broadness becomes lower in the sense of first-order stochastic dominance ($\tilde{F}_D^* \geq_{FOSD} \tilde{F}_H^*$); and the platform profit increases ($\Pi_H^* > \Pi_D^*$).*

Proof. Clearly, $r_D^* \geq r_H^*$ by Lemma E.3 because the pure discovery platform sets $r_D^* = \bar{r}$ to maximize $\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, 1)$. Then,

$$\tilde{F}_D^* = \tilde{F}(z; r_D^*, 1) \geq_{FOSD} \tilde{F}(z; r_H^*, \tau_H^*) = \tilde{F}_H^*$$

by Lemma E.3. Meanwhile, the profit comparison follows from the profit replication argument as in Proposition 3. ■

Proposition E.2. *Consider a move from the pure membership mode to the hybrid mode. Then, in the model of asymmetric creators described in this section, there exist weakly positive thresholds A_{mono}, A'_{mono}, such that:*

- *the equilibrium distribution of content design broadness becomes higher in the sense of first-order stochastic dominance ($\tilde{F}_H^* \geq_{FOSD} \tilde{F}_M^*$) if $A \geq A'_{mono}$;*
- *The platform profit increases if and only if $A \geq A_{mono}$; Moreover, $A_{mono} > 0$ if G is sufficiently inelastic.*

Proof. We first note that either $r_H^* = \underline{r}$ or $\tau_H^* = 1$ (or both). To see this, suppose by contradiction, suppose $r_H^* > \underline{r}$ and $\tau_H^* < 1$. By the supposition and strict concavity of $\tilde{u}$, we know $\frac{d}{dr} \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r_H^*, \tau_H^*)$ is strictly increasing in r. Then, the platform can increase τ while lowering r simultaneously to make the distribution of equilibrium design at $\tilde{F}(z; r_H^*, \tau_H^*)$ (Lemma E.3), while strictly increasing its per-viewer revenue $\tau \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\tilde{F}(z; r_H^*, \tau_H^*)$, meaning (r_H^*, τ_H^*) is not optimal, a contradiction.

If $\tau_H^* = 1$, then $\tilde{F}_H^* = \tilde{F}(z; r_H^*, \tau_H^*) \geq_{FOSD} \tilde{F}(z; -\infty, \tau_M^*) = \tilde{F}_M^*$ by Lemma E.3. Suppose instead $\tau_H^* < 1$, then by the previous paragraph we know $r_H^* = \underline{r}$. Therefore, τ_H^* is characterized by FOC

$$\begin{aligned} 0 \quad = \quad & \frac{G\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; \underline{r}, \tau)\right)}{g\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; \underline{r}, \tau)\right)} \left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\tilde{F}(z; \underline{r}, \tau) + \tau \frac{d}{d\tau} \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\tilde{F}(z; \underline{r}, \tau)\right) \\ & + (A + \tau v(\lambda)) \underbrace{\frac{d}{d\tau} \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; \underline{r}, \tau)}_{\geq 0 \text{ by Lemma } E.3}, \end{aligned}$$

where the LHS is increasing in A. Thus, τ_H^* is increasing in A by the implicit function theorem, whereas τ_M^* is independent of A. The intermediate value theorem establishes the existence of threshold A'_{mono} such that $\tau_H^* \geq \tau_M^*$, in which case we have $\tilde{F}_H^* = \tilde{F}(z; \underline{r}, \tau_H^*) \geq_{FOSD} \tilde{F}(z; -\infty, \tau_M^*)$ by Lemma E.3.

Meanwhile, the profit comparison follows from the same envelope theorem argument as in the proof of Proposition 2. To show $A_{mono} > 0$ if G is sufficiently inelastic, let $A = 0$ and $G(.)$ be a constant, then

$$\begin{aligned} \Pi_H^* \quad & = \quad G \times \tau_H^* \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\tilde{F}(z; r_H^*, \tau_H^*) - C \\ & < \quad G \times \tau_H^* \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\tilde{F}(z; -\infty, \tau_H^*) - C \\ & \leq \quad G \times \tau_M^* \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\tilde{F}(z; -\infty, \tau_M^*)) = \Pi_M^*. \end{aligned}$$

where the first inequality follows from $v(.)$ being decreasing Lemma E.3, and the second inequality follows from the definition of τ_M^* and $C \geq 0$. ■

F Extension: elastic creator participation and network effects

F.1 Summary

In our model, all creators are active and join the platform in the equilibrium. As such, participation by consumers and creators are essentially independent, as long as we rule out the coordination problem associated with the trivial equilibrium with no participation. In that sense, our model does not feature explicit cross-group network effects emphasized by the literature of two-sided markets (Rochet and Tirole 2003; Armstrong 2006), except in the form of participation coordination. Allowing for elastic creator participation (that is, platform decisions affect the mass of active creators in a continuous manner) does not affect our results so long as a strictly positive mass of creators are always active. This is due to the assumptions of: (i) a continuum of symmetric creators and (ii) unit demand by consumers.

In this section, we expand the asymmetric creators model of Section E by exploring the impact of elastic creator participation. We assume that creators face a fixed cost $c > 0$ for being active (regardless of whether the creator is joining the discovery portal, the membership portal, or both) so that their participation is elastic. In this case, participation decisions of consumers and creators are interdependent, thus generating cross-group network effects.

To see this point, let us focus on the most general case of a hybrid mode. It can be shown that there exists a unique creator participation threshold $\hat{a} \in [\underline{a}, \bar{a}]$ such that all creators with $a_i \geq \hat{a}$ are active whereas those with type $a_i < \hat{a}$ are inactive. The marginal creator with type $a_i = \hat{a}$ is indifferent between being active (and joining the monopoly platform) and being inactive:

$$G\left(\int_{\lambda(\hat{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, \tau, \hat{a})\right)\left(\frac{\exp\left(\frac{\tilde{u}(\lambda(\hat{a}))}{1-r}\right)}{\int_{\hat{a}}^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)}\right) \lambda(\hat{a})(\hat{a} + (1-\tau)v(\lambda(\hat{a}))) = c, \tag{F.1}$$

where $\tilde{F}(z; r, \tau, \hat{a})$ is the effective distribution of content design, conditioned on the set of participating creators:

$$\tilde{F}(z; r, \tau, \hat{a}) = \frac{\int_{\lambda(\hat{a})}^{z} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i}{\int_{\lambda(\hat{a})}^{\lambda(\bar{a})} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i} \quad \text{for} \quad z \in [\lambda(\hat{a}), \lambda(\bar{a})].$$

Notice from (F.1) that creator participation depends on consumer participation $G(.)$, which in turn depends on creator participation threshold $\hat{a}$ through the distribution of content design $\tilde{F}(z; r, \tau, \hat{a})$. If $c \to 0$ then we recover the model in Section E.1.

For each given r and τ chosen by the platform, let $\hat{a} = \hat{a}(r, \tau)$ denote the solution of (F.1). Then, we prove that if buyer participation (as measured by distribution G) is not too elastic, then (r, τ) is increasing in r and τ. That is, fewer creators participate when the platform intensifies the competition for recommendations or increases its fee. Taking into account this change in creator participation, it can be shown that $\tilde{F}(z; r, \tau, \hat{a}(r, \tau))$ is increasing in τ and r in the sense of first-order stochastic dominance (FOSD), as in Lemma E.1.

Based on this result, we verify that the results in Proposition 2 and Proposition 3 remain valid. The only mechanical difference in this scenario is that the platform's choice of mode now influences the market outcome through an additional effect of changing the composition of a_i of the active creators.

F.2 Analytical details and proofs

We apply the same set of assumptions as in Section E. Recall creators are asymmetric in their intrinsic image gain $a_i \in [\underline{a}, \bar{a}]$, following distribution F. Let $\hat{a} \in [\underline{a}, \bar{a}]$ be the threshold such that all creators with $a_i \geq \hat{a}$ are active (and join the platform) whereas those with $a_i < \hat{a}$ are inactive. Conditioned on the set of participating creators $a_i \geq \hat{a}$, denote the recommendation-weighted distribution of content design as

$$\tilde{F}(z; r, \tau, \hat{a}) = \frac{\int_{\lambda(\hat{a})}^{z} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i}{\int_{\lambda(\hat{a})}^{\lambda(\bar{a})} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i} \quad \text{for} \quad z \in [\lambda(\hat{a}), \lambda(\bar{a})],$$

where recall $f_\lambda(z) = f(\lambda^{-1}(z)) \frac{\partial \lambda^{-1}(z)}{\partial z}$ and

$$\lambda(a_i) = \arg\max_{\lambda_i \in [0,1]} \left\{ \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) \times \lambda_i \times (a_i + (1-\tau) v(\lambda_i)) \right\}.$$

Observe that the distribution $\tilde{F}(z; r, \tau, \hat{a})$ becomes higher in the sense of FOSD when the threshold $\hat{a}$ increases:

$$\frac{\partial \tilde{F}(z; r, \tau, \hat{a})}{\partial \hat{a}} = -\left(\int_{z}^{\lambda(\hat{a})} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i \right) \exp\left(\frac{\tilde{u}(\lambda(\hat{a}))}{1-r}\right) f_\lambda(\lambda(\hat{a})) \frac{\partial \lambda(\hat{a})}{\partial a} < 0$$

For any given r and τ, the threshold type $\hat{a}$ is pinned down by zero-profit condition

$$G\left(\int_{\lambda(\hat{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, \tau, \hat{a}) \right) \left(\frac{\exp\left(\frac{\tilde{u}(\lambda(\hat{a}))}{1-r}\right)}{\int_{\hat{a}}^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)} \right) \lambda(\hat{a}) (\hat{a} + (1-\tau) v(\lambda(\hat{a}))) = c. \tag{F.2}$$

As $\int_{\lambda(\hat{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, \tau, \hat{a})$ and $1/\int_{\hat{a}}^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)$ are increasing in $\hat{a}$, by the envelope theorem we conclude that the left-hand side of (F.2) is increasing in $\hat{a}$. Thus, the solution $\hat{a} \in [\underline{a}, \bar{a}]$ must be unique whenever it exists. If the solution $\hat{a} \in [\underline{a}, \bar{a}]$ to (F.2) does not exist, then we either set $\hat{a} = \underline{a}$ (all creators are active) or $\hat{a} = \bar{a}$ (all creators are inactive).

Lemma F.1. *Consider $\hat{a}$ implicitly defined by (F.2). If G is sufficiently inelastic, then $\hat{a} = \hat{a}(r, \tau)$ is increasing in r and τ. Consequently, $\tilde{F}(z; r, \tau, \hat{a}(r, \tau))$ is increasing in τ and r in the sense of first-order stochastic dominance (FOSD).*

Proof. Denote $\tilde{\pi}(a) = \exp\left(\frac{\tilde{u}(\lambda(a))}{1-r}\right) \lambda(a) (a + (1-\tau) v(\lambda(a)))$ and the left-hand side of (F.2) as

$$\phi(a) \equiv G\left(\int_{\lambda(a)}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, \tau, a) \right) \frac{\tilde{\pi}(a)}{\int_{a}^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)}$$

for arbitrary a. By the implicit function theorem,

$$\frac{d\hat{a}}{d\tau} = \frac{\frac{\partial \phi(a)}{\partial \tau}}{-\frac{\partial \phi(a)}{\partial a}} |_{a=\hat{a}}.$$

We already know that

$$\begin{aligned} \frac{\partial \phi(a)}{\partial a} &= g\left(\int_{\lambda(a)}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, \tau, a) \right) \frac{\tilde{\pi}(a)}{\int_{a}^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)} \frac{\partial}{\partial a} \int_{\lambda(a)}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, \tau, a) \\ &+ G\left(\int_{\lambda(a)}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z; r, \tau, a) \right) \frac{\partial}{\partial a} \left(\frac{\tilde{\pi}(a)}{\int_{a}^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)} \right) > 0 \end{aligned}$$

for any a. Meanwhile,

$$\frac{\partial \phi(\hat{a})}{\partial \tau} = g\left(\int_{\lambda(a)}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z;r,\tau,a)\right) \frac{\tilde{\pi}(a)}{\int_a^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)} \frac{\partial}{\partial \tau} \int_{\lambda(a)}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z;r,\tau,a)$$
$$+G\left(\int_{\lambda(a)}^{\lambda(\bar{a})} \tilde{u}(z) d\tilde{F}(z;r,\tau,a)\right) \frac{\partial}{\partial \tau} \left(\frac{\tilde{\pi}(a)}{\int_a^{\bar{a}} \exp\left(\frac{\tilde{u}(\lambda(a_i))}{1-r}\right) dF(a_i)}\right),$$

where the second line is strictly negative due to the Envelope Theorem (so that $\partial\tilde{\pi}/\partial\tau \leq 0$) and $\lambda(a_i)$ being increasing in τ. Therefore, if G is sufficiently inelastic, i.e., $\frac{g(.)}{G(.)}$ is small, then $\frac{\partial \phi(\hat{a})}{\partial \tau} < 0$. We then conclude $\frac{d\hat{a}}{d\tau} > 0$. The same proof applies for showing $\frac{d\hat{a}}{dr} > 0$.

Then, to establish first-order stochastic dominance:

$$\frac{d\tilde{F}(z;r,\tau,\hat{a}(r,\tau))}{d\tau} = \frac{\partial \tilde{F}(z;r,\tau,\hat{a}(r,\tau))}{\partial \tau} + \frac{\partial \tilde{F}(z;r,\tau,\hat{a}(r,\tau))}{\partial a} \frac{d\hat{a}(r,\tau)}{d\tau} < 0.$$

by Lemma E.3 and the result above. The same step applies for showing $\frac{d\tilde{F}(z;r,\tau,\hat{a}(r,\tau))}{dr} < 0$. ■

Intuitively, the sign of $d\hat{a}/d\tau$, which reflects how the increase in τ affects the *composition of active creators*, is generally ambiguous due to two opposing effects. On the one hand, a higher τ reduces each creator's per-viewer profit, thus raising $\hat{a}$ as fewer creators participate. On the other hand, a higher τ induces a greater content broadness, thus raising the mass of consumers who initiate search. This market expansion effect increases the marginal creator's profit, thus lowering $\hat{a}$. Nonetheless, if G is sufficiently inelastic, then the first effect dominates.

We are now ready to compare across the three modes of operations. For simplicity, denote distribution

$$\hat{F}(z;r,\tau) = \tilde{F}(z;r,\tau,\hat{a}(r,\tau))$$
$$= \frac{\int_{\lambda(\hat{a}(r,\tau))}^{z} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i}{\int_{\lambda(\hat{a}(r,\tau))}^{\lambda(\bar{a})} \exp\left(\frac{\tilde{u}(\lambda_i)}{1-r}\right) f_\lambda(\lambda_i) d\lambda_i} \quad \text{for} \quad z \in [\lambda(\hat{a}(r,\tau)), \lambda(\bar{a})].$$

The profit function of the monopoly platform in the pure membership, pure discovery, and hybrid modes are, respectively,

$$\Pi_M(\tau) = G\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\hat{F}(z;-\infty,\tau)\right) \tau \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\hat{F}(z;-\infty,\tau);$$
$$\Pi_D(r) = G\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\hat{F}(z;r,1)\right) A - C;$$
$$\Pi_H(r,\tau) = G\left(\int_{\lambda(\underline{a})}^{\lambda(\bar{a})} \tilde{u}(z) d\hat{F}(z;r,\tau)\right) \left(A + \tau \int_{\lambda(\underline{a})}^{\lambda(\bar{a})} v(z) d\hat{F}(z;r,\tau)\right) - C$$

Denote Π_M^*, Π_D^*, and Π_H^* as the respective maximized profit, and the corresponding effective distribution of content design as $\hat{F}_M^*$, $\hat{F}_D^*$, and $\hat{F}_H^*$.

Proposition F.1. *Consider a move from the pure discovery mode to the hybrid mode and suppose G is sufficiently inelastic. Then, in the model of asymmetric creators with elastic creator participation described in this section, the equilibrium distribution of content design broadness becomes lower in the sense of first-order stochastic dominance ($\hat{F}_D^* \geq_{FOSD} \hat{F}_H^*$); and the platform profit increases ($\Pi_H^* > \Pi_D^*$).*

Proof. The results follow from the same proof as Proposition E.1 after applying Lemma F.1. ■

Proposition F.2. *Consider a move from the pure membership mode to the hybrid mode and suppose G is sufficiently inelastic. Then, in the model of asymmetric creators with elastic creator participation described in this section, there exist weakly positive thresholds A_{mono}, A'_{mono}, such that:*

- *the equilibrium distribution of content design broadness becomes higher in the sense of first-order stochastic dominance ($\hat{F}_H^* \geq_{FOSD} \hat{F}_M^*$) if $A \geq A'_{mono}$;*
- *The platform profit increases if and only if $A \geq A_{mono} > 0$.*

Proof. The results follow from the same proof as Proposition E.2 after applying Lemma F.1. ■